Applied Mechanics Reviews

A Critical Review of the World Literature in Applied Mechanics

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Abbreviations: Title abbreviations of periodicals follow the World List of Scientific Periodicals. Barring other indications, volume number, page numbers, and date are shown in this order. Scientific terms are abbreviated according to American Standards Association.

Russian Transliteration: Excepting traditionally established spellings, the doubt-raising Russian letters will be substituted according to the following table representing a compromise between pronunciation and unique determination of the original spelling:

в	г	е	ё	ж	и	й	у	х	ц	ч	ш	щ	ъ	ы	ь	э	ю	я
v	g	e	yo	zh	i	ĭ	u	kh	ts	ch	sh	shch	″	ĭ	′	è	yu	ya

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Communications

Reviewers wanted

APPLIED MECHANICS REVIEWS does not have specialized reviewers for papers in

Russian, on boundary layer; heat transfer; combustion and flame propagation; diabatic flow; thermodynamics of engines;

Swedish, on wind tunnels;

Spanish, on structures.

We appeal to our readers for offers of their services or suggestions for names. Ed.

Invitation to volunteer reviews

Our reviewers occasionally take the risk of sending in an unrequested review. AMR warmly welcomes such generous initiative, even though, for various reasons, we may be regretfully unable to use the manuscript. If your unsolicited review reaches us within about one month after our receipt of the original material reviewed, the chances are excellent that it will be used. Ed.

Theoretical and Experimental Methods

(See also **Revs. 1028, 1029, 1038, 1042, 1049, 1051, 1067, 1068, 1100, 1154, 1155**)

1011. Author's guide for preparing manuscript and handling proof, New York, John Wiley and Sons, Inc.; London, Chapman and Hall, Ltd., 1950, xvi + 80 pp. Cloth, 6 × 9 in., $2.

This manual is intended for authors without previous experience in book production, and its use will save them a great deal of time and many unnecessary misunderstandings. Ed.

1012. Nathan Grier Parke III, Guide to the literature of mathematics and physics including related works on engineering science, New York and London, McGraw-Hill Book Co., 1947, xv + 205 pp. Cloth, 6 × 9 in., $5.

Part I (General considerations) consists of sections on the principles of reading and study, self-directed education, literature search and periodicals, and contains Columbus-egg-type advice, well-known to mature research workers, but frequently revealing to the beginner. ("The author has had the experience of going to considerable trouble to get information he did not realize was in his dictionary.")

Part II lists about 2300 titles under 150 subject headings. The list is preceded by a classified arrangement of subject headings, and closes with an author and subject index. Each heading in the list is followed by a brief description. English titles account for the bulk of the entries, German and French come next, and a few Italian and one Polish title have been spotted. Ed.

1013. Georgio Marolli, Technical dictionary, English-Italian; Italian-English (Dizionario tecnico, Inglese-Italiano; Italiano-Inglese), Florence, Felice Le Monnier, 1950, 630 pp. + 16 plates. Paper, 5 × 7 in., $4.36.

This second edition shows many improvements with respect to the first edition which was published only a few months ago. A supplement is added at the end of the volume and some illustrative tables complete the work. The absence from the market of an up-to-date technical dictionary of the English and Italian languages makes the new edition very valuable.

Enrico Volterra, USA

1014. Hans Straub, History of civil engineering (Die Geschichte der Bauingenieurkunst), Basel, Verlag Birkhäuser, 1949, 285 pp., 79 figs. Cloth, 6 × 9.8 in., $5.18.

There are some books on the history of architecture in which the historical development of the construction of such engineering structures as bridges and domes is treated; an integrated and comprehensive history of civil engineering has, however, yet to be written.

The present book is an attempt to write such a history in less than 300 pages, with particular emphasis on the development of structural mechanics. The nine chapters of the book deal with: civil engineering of antiquity, of the middle ages (especially the Romanic and Gothic domes), and of the Renaissance, with the fortification and bridge engineers in Italy and France; the development of structural mechanics from Leonardo da Vinci and Galilei to Coulomb and Navier, and later to Ritter and Mueller-Breslau; the development of engineering materials and methods of construction in the 19th century, including short surveys of the development of hydraulic engineering, and of the use of machines in building operations. The book closes with a survey of present-day problems of structural engineering, which occasionally is not more than a list of names of engineers, and includes a short discourse on the interrelation of civil engineering and architecture, which points to the necessity in modern engineering construction of a synthesis of methods of rational design with an intuitive feeling for form.

Because of the size of the book the treatment of all subjects is necessarily very sketchy; only the presentation of the development of structural mechanics goes occasionally into greater detail.

A. M. Freudenthal, USA

1015. Guido Ascoli, Some synthetic remarks about integrating instruments (in Italian), Rend. Sem. Mat. Fis. Milano 18 (1947), 36–54 (1948).

This is an attempt at a unified theory of certain planimeters and integraphs. A planimeter is defined as a mechanical analog device for determining a double integral over the area enclosed by a curve by tracing the curve with a pointer. Such a device is necessarily a nonholonomic mechanical system and therefore contains as constituent at least a rolling wheel. The theory of an instrument containing such a rolling wheel is discussed under rather general assumptions and then specialized. The key tool is Green's theorem for a plane area. There is a briefer discussion of integraphs, i.e., devices for drawing the graph of ${}_a\int^x f(x)dx$ by tracing that of $f(x)$.

Courtesy of Mathematical Reviews H. B. Curry, USA

1016. L. A. Lyusternik and V. A. Ditkin, Approximate formulas for the calculation of multiple integrals (in Russian), Izv. Akad. Nauk SSSR no. 8, 1163–1168 (1948).

Another version of an article already reviewed [Doklady Akad. Nauk SSSR 61, 441–444 (1948); Rev. 3, **811**].

Courtesy of Mathematical Reviews E. M. Bruins, Holland

1017. Warren S. Loud, On the long-run error in the numerical solution of certain differential equations, J. Math. Phys. 28, 45–49 (Apr. 1949).

The equation $y' = ay$ and certain types of approximate solutions evaluated at n equally spaced points of a basic interval are compared, and a scheme for computing a bound to the "relative error" between the dominant term of an approximate solution and the exact solution is indicated. Several error terms, dismissed as negligible for "large values of n," make the estimates proposed of dubious value for quantitative error control. Extension to linear equations with constant coefficients is indicated.

L. B. Hedge, USA

1018. R. H. J. Germay, On a successive approximations method for integration of linear systems of differential equations: extension to normal systems of a general form (in French), Bull. Soc. Roy. Sci. Liége 16, no. 6, 119–125 (1947).

The author modifies the standard method of successive approximations for the system $y_j' = f_j(x, y_1, \ldots, y_n)$, where $j = 1, \ldots, n$, so that $y_{j,m}(x)$, the mth approximation for y_j, is defined in terms of $y_{1,m}, \ldots, y_{j-1,m}, y_{j,m-1}, \ldots, y_{n,m-1}$ instead of $y_{1,m-1}, \ldots, y_{n,-m}$.

Courtesy of Mathematical Reviews P. Hartman, USA

1019. G. P. Akilov, On the application of a method of solution of nonlinear functional equations to the investigation of systems of differential equations (in Russian), Doklady Akad. Nauk SSSR 68, 645–648 (1949).

Kantorovich has investigated the solution of an equation $P(x) = 0$ where P is a certain nonlinear operator [see Doklady Akad. Nauk SSSR 59, no. 7 (1948); Uspekhi Mat. Nauk 6, p. 89 (1948)]. His methods and results, based essentially upon a method of successive approximation, are applied here to the solution of a "diagonal" system over $0 \leq t \leq 1$:

$$dx_s/dt = F_s(t,x_s,\ldots,x_n); \quad (s = 0, 1,\ldots,n)$$

with boundary conditions $\varphi_s(x_0^0,\ldots, x_n^0;\ x_0^1,\ldots, x_n^1) = 0$, $x_i^0 = x_i(0)$, $x_i^1 = x_i(1)$, where F_s and φ_s are sufficiently continuous in the variables other than t.

Periodic solutions are also discussed. Extensions are made to the infinite t interval $[0, \infty]$ with applications to Liapounoff stability and also to the quasilinear problems considered by Krylov-Bogolyubov [*Introduction to nonlinear mechanics* (1937); also its (partial) translation as Ann. Math. Studies no. 11]. The results only are described and proofs will (presumably) be given later.

S. Lefschetz, USA

1020. E. C. Titchmarsh, On the uniqueness of the Green's function associated with a second-order differential equation, Canadian J. Math. 1, 191–198 (1949).

Let $G(x,\xi,\lambda)$ be the Green's function for the differential equation

$$(*) \quad d^2\varphi/dx^2 + [\lambda - q(x)]\varphi = 0$$

on the interval $(0, \infty)$. It has the following properties: (1) If $x \neq \xi$, $G(x,\xi,\lambda)$ has continuous partial derivatives with respect to x up to the second order, and satisfies (*) as a function of x; (2) for some real value of α,

$$G(0,\xi,\lambda) \cos \alpha + G_x(0,\xi,\lambda) \sin \alpha = 0;$$

(3) G belongs to $L^2(0, \infty)$ as a function of x; (4) $G(x,\xi,\lambda)$ is continuous at $x = \xi$, while $G_x(\xi+, \xi,\lambda) - G_x(\xi-,\xi,\lambda) = -1$. It is proved that if $q(x) \geqq -Ax^2 - B$, where A and B are nonnegative constants, then the Green's function is unique. A similar result is established for the equation

$$\nabla^2\varphi + [\lambda q(x,y)]\varphi = 0,$$

the region being the whole plane.

Courtesy of Mathematical Reviews H. Pollard, USA

1021. D. G. Bourgin, Two problems of mixed type for the damped wave equation, Quart. appl. Math. 6, 279–299 (1948).

This paper deals with the solution of "mixed" problems for the damped wave equation (A) $z_{tt} - z_{xx} + z = 0$, which are similar to a problem solved recently by S. Chandrasekhar [Proc. Cambridge philos. Soc. 42, 250–260 (1946)]: z is to be determined in the unbounded region bounded by a segment $0 \leqq x \leqq l$ of the x-axis and two "time-like" half-lines P and P'. Here P consists of the points of the line $t + \lambda x = 0$ with positive t, where $\lambda > 1$. In one of the two problems considered, the half-line P' is given by $x = l$, $t>0$; in the other problem by $t + \lambda x = \lambda l$, $t>0$. The solution z of (A) to be determined is subject to the boundary conditions $z = z_t = 0$ for $0 \leqq x \leqq l$, $t = 0$; $z_\nu = 0$ on P'; $z_\nu = hz + \psi(x,t)$ on P, where h is a positive constant and ψ a given function. Here z_ν denotes the "conormal" derivative of z, formed with respect to the hyperbolic metric with line element $ds = (dt^2 - dx^2)^{1/2}$, which is the metric associated with the characteristic form of the differential equation (A).

The problem of determining z is reduced to the solution of an integral equation for z along P. In obtaining this integral equation from Green's identity, extensive use is made of the "method of images," where here, however, the images are not the images formed by ordinary Euclidean reflection, but are taken with respect to the hyperbolic metric mentioned above. The integral equation is solved explicitly in terms of quadratures with the help of Laplace transforms.

Courtesy of Mathematical Reviews F. John, USA

1022. H. B. Mann, Analysis and design of experiments, New York, Dover Publications, 1949, x + 195 pp. Cloth, 5 × 8 in., $2.95.

Most textbooks and manuals in statistics contain at least some chapters on systematic design of experiments and interpretation of data (analysis of variance and covariance). As was the case in the original works by R. A. Fisher and F. Yates, the methods are now usually motivated on intuitive grounds in the literature, and an extensive description from the viewpoint of modern mathematical statistics has been lacking. The present book fills this gap.

Chapters I and II give a mathematical introduction. The case of one-way classification is treated in the classical manner in chapter III. Chapter IV contains the general theory of testing of linear hypotheses, and is the central part of the first half of the book. It is logically followed by the case of r-way classification (chapter V) and an analysis of the statistical properties of the tests (chapter VI). Experimental design is treated in chapters VII–IX and XI–XII. Chapter X is an extension of chapter V to nonorthogonal data. The analysis of covariance is given two pages in chapter XIII. Interblock estimates and interblock variances are treated in chapter XIV. Tables of the F-distribu-

tion and of the power function of the analysis of variance test complete the book.

Since the book is very concentrated, it will not be easy to read even for a mathematician or an advanced student of statistics, according to this reviewer's belief. The practical statistician or experimenter will undoubtedly have a difficult task in comprehending the book, especially as no numerical examples are treated. The reader is given a clear picture of the assumptions on which the methods are based. On the other hand, nothing is said about how rigorously these assumptions need to be fulfilled in practice, or about useful transformations.

Unfortunately, the book contains many misprints.

Nils G. Blomqvist, Sweden

Mechanics (Dynamics, Statics, Kinematics)

(See also Revs. 1035, 1036, 1041, 1066, 1117)

1023. **Arnold Sommerfeld, Lectures on theoretical physics, vol. II: Mechanics (Vorlesungen über theoretische Physik, Band II: Mechanik)**, Wiesbaden, Dieterich'sche Verlagsbuchhandlung, 1947, xv + 375 pp., 74 figs. Paper, 9 × 6 in., \$3.60.

The stated purpose of this book, to develop in some completeness the mathematical methods used in physics, is accomplished incidentally in an inclusive, penetrating, and orderly exposition of the elements of mechanics. The style is reminiscent of Kirchhoff's lectures: the general principles once explained, examples of direct physical interest are solved mathematically and the results are briefly compared with experiment. Three-fourths concern hydrodynamics (including masterful brief surveys of the theory of vortexes and the theory of waves); the remainder, hydrostatics and elasticity. Many recent results are worked into this classical treatment of a classical subject (e.g., the discussion of turbulence in §38). Except for a few illustrative parentheses containing supplementary results from molecular theories, the viewpoint is consistently phenomenological. The mathematical level is high for a current physics book, with one exception: while the author emphasizes the importance of invariance, the discussions of it are awkward and excessive, and neither vector nor tensor methods are fully exploited. The historical notes, while often capriciously selected, inaccurate, and misleading, are sometimes original and interesting (e.g., §19).

Among many details new to this reviewer may be noted the following: a beautiful experiment illustrating the impossibility of hydrostatic equlibrium when the force potential is many-valued, and the discontinuity in pressure consequent upon rendering a many-valued potential single-valued by the imposition of a cut (§6); a proof that MacCullagh's quasi-elastic theory of the luminiferous ether is isomorphic to Maxwell's electromagnetic theory in two different ways (§15); consistent use of Newton's formula C^2 = (elasticity/density) for all cases of wave motions, with suitable definition of "elasticity"; use of the known but seldom mentioned similarity parameter $S \equiv p/\rho v^2$ (which can be used for incompressible and compressible fluids alike, but for the latter is conveniently replaced by the Mach number) to show that the pressure-drop formulas for both laminar and turbulent flow in a pipe may be formally reconciled.

C. A. Truesdell, USA

1024. **B. Davídov, Variational principle and canonical equations for an ideal fluid** (in Russian), Doklady Akad. Nauk SSSR 69, 165–168 (1949).

The author obtains a simple variational principle by varying ρ and v_k in the Lagrangian (1) $L = \frac{1}{2}\rho v_k^2 - \epsilon(\rho)$, where $\epsilon'(\rho) = \int dp/\rho$ is the density of the potential energy. If one assumes only that the equation of continuity (2) $\dot{\rho} + \Sigma(\rho v_k)_k = 0$ holds, the equations for a vortex-free flow are obtained.

In order to get the equations for a vortical flow, the author introduces a single coordinate α for each particle, this coordinate satisfying the condition (3) $\dot{\alpha} + \Sigma\alpha_k v_k = 0$.

By requiring that (3) hold and that the values of α at the initial and terminal instants remain unchanged while variation takes place, the author fixes one coordinate of each particle at these instants. Adding the left-hand sides of (2) and (3) to the Lagrangian (1), after multiplying them by undetermined multipliers $-\lambda$ and ϕ (considered to be functions of x_k and t) respectively, and varying in the obtained expression all quantities independently, the desired equations follow in the usual way. The canonical equations for λ, α and ρ, ϕ are found to be identical with the Lagrangian equations obtained above.

E. Leimanis, Canada

1025. **K. P. Persidskiĭ, Uniform stability in the first approximation** (in Russian), Prikl. Mat. Mekh. 13, 229–240 (1949).

The author considers the infinite system

$$dx_i/dt = f_i(x_1, x_2, \ldots, x_n, \ldots),$$

where

$$f_i = {}_j\Sigma\, p_{ij}x_j + g_i(x),$$

$$|g_i(x)| \leq \|x\|\, g(\|x\|),\quad \|x\| = \sup_i(|x_1|, |x_2|, \ldots),\; g(\|x\|) \to 0$$

as $\|x\| \to 0$. Ordinary stability of the particular solution $x_1 = x_2 = \ldots = 0$ requires the existence of an r, depending upon ϵ and in general t_0, such that $\|x(t_0)\| \leqq r$ implies $\|x(t)\| \leq \epsilon$ for $t = t_0$. If r is independent of t_0, the stability is said to be uniform. The stability is asymptotic if $\lim_{t\to\infty} x_i(t) = 0$ for all i. Various theorems are demonstrated illustrating the connection between the uniform and asymptotic stability of the solution of the linear approximation and that of the solution of the full equation. In particular, a necessary and sufficient condition for uniform stability is given.

R. Bellman, USA

1026. **I. M. Volk, On a sufficient condition for the stability of a motion in the critical case of two roots with vanishing real parts** (in Russian), Prikl. Mat. Mekh. 13, 459–462 (1949).

For the autonomous system

$$dx/dt = X,\; dy/dt = Y,\; dx_i/dt = \Sigma p_{ij}x_j + X_i;\; i,j = 1,2,\ldots,n,$$

there is given a condition of stability at the origin, and existence of periodic motion near it. The condition stipulates, among many other things, that the eigenvalues of the matrix p_{ij} have negative real parts, and that an integral $H(x,y;\, x_1,\ldots,x_n)$, independent of t exists. Explicit determination of the period is given. [See also Rev. 3, **211**].

Ed.

1027. **Maurice Parodi, A remark on stability; An application of a Hadamard theorem to stability theory; A complement to a paper on stability** (in French), C. R. Acad. Sci. Paris 228: 51–52; 807–808; 1198–1200 (1949).

The first paper derives, from a well-known theorem on determinants, the sufficient condition: $a_{ii} > 0$ and $>_k \Sigma|a_{ik}|$, $k \neq i$, for the eigenvalues of a real matrix a_{ik} to have no positive real parts.

The second paper [see also Rev. 2, **1098**] uses the same theorem to derive a sufficient condition for the equation

$$|a_{ij}| = 0,\; a_{ii} = (a_i + b_i z)/(\alpha_i + \beta_i z),\; a_{ij},\, a_i,\, b_i,\, \alpha_i,\, \beta_i \text{ real},$$

to have all roots with negative real parts.

The third paper shows that the condition of the first paper assures, in certain cases, that the eigenvalues avoid a neighborhood of zero.

Ed.

1028. Walter Wuest, The accuracy of transmission by linkages and roller coupling in instruments (in German), Technik 3, 305–319 (July 1948).

In the construction of measuring apparatus, linkage mechanisms are used in many cases for transmitting the angular displacement of the detector to the indicator. These displacements are generally very small. The mechanisms used should realize a constant angular ratio u during the transmission.

The author studies the deviations from a constant given ratio u for mechanisms where the cranks are connected by a link (three-bar linkages) or by roller contact. The deviations are expressed approximatively in terms of the second and third orders. As it is possible to annul the terms of second order, the precision of the transmission ratio can be indicated by the third-order terms. D. De Meulemeester, Belgium

1029. Th. de Donder and F. van den Dungen, On relative motion of rigid bodies (in French), C. R. Acad. Sci. Paris 228, no. 3, 221–223 (Jan. 1949).

Let O, X^a $(a = 1,2,3)$ be an initial trihedron. The motion of a solid entraining a trihedron G, ξ^a is given by $X^a = H_b{}^a\xi^b + X_G{}^a$. The motion is defined by the equations $\delta L/\delta X_G{}^a = 0$, $\delta L/\delta H_b{}^a = 0$, where L is the Lagrange function, and δ the variational derivative. If o, x^a is another initial trihedron, $x^a = h_b{}^a\xi^b + x_g{}^a$. The authors show that $\delta L^z/\delta x_G{}^a = 0$, $\delta L^z\delta h_b{}^a = 0$, if L^z is the Lagrange function of the relative motion.

Courtesy of Mathematical Reviews O. Bottema, Holland

1030. V. Valcovici, The wrench on a rigid body in a field of centrifugal forces (in French), Bull. Sec. sci. Acad. roum. 30, 70–74 (1947).

The author shows that the forces acting on a body rotating about a fixed axis reduce in general to a wrench of which the central axis is normal to the axis of rotation, and lies in the plane containing the latter and the center of gravity of the solid. If the axis of rotation contains a point with respect to which it is a principal axis, then the wrench reduces to a force through this point. Thus, only if this point coincides with the projection of the center of gravity upon the axis of rotation, does the force pass through the center of gravity. P. C. Dunne, England

1031. H. W. Swift, The dynamics of crank-press operation, Engineering 168, no. 4376, 605–608 (1949).

This paper shows some mathematical analyses of crank-press forces for different amounts of interference or lack of clearance between the bottommost position of the movable crosshead and the workpiece. However, it would be highly desirable to show more completely the correlation between measured and calculated forces. R. G. Sturm, USA

Gyroscopics, Governors, Servos

(**See also Revs. 1030, 1116**)

1032. Charles Fox, The mechanical determination of position and velocity on the earth's surface, Proc. Cambridge philos. Soc. 45, 311–315 (Apr. 1949).

The author considers the problem of determining his position and velocity on the surface of the earth by making observations on local instruments alone. The solution of the problem, if made feasible, is of importance to a submariner for, in general, he has no means of making regular celestial observations.

The author suggests the use of four instruments: a gyrocompass, a vertical indicating gyro, a gyro having an axis which can rotate in the horizontal plane, and a gyro having an axis which can rotate in a vertical plane which is normal to gyronorth (a direction established by the first instrument). Two moments are measured: M_1 which is the torque required to hold the axis of the third gyro at right angles to gyronorth, and M_2 which is required to hold the axis of the fourth gyro horizontal.

Let λ, μ, n, w, and α be respectively the latitude, longitude, northerly component of velocity, westerly component of velocity, and the angle between gyronorth and true north. The author proposes that a computer be built which will solve a set of five simultaneous differential equations, involving the five unknown variables listed, in which M_1 and M_2 are inputs, and known values of λ and μ at some instant are boundary values.

Since this is only a proposal, it is yet to be demonstrated that the two moments M_1 and M_2 can be indicated with sufficient precision to make the proposal feasible. Horace M. Trent, USA

1033. I. Flügge-Lotz and K. Klotter, On the motions of an oscillating system under the influence of flip-flop controls, Nat. adv. Comm. Aero. tech. Memo. no. 1237, 57 pp. (1949).

This paper analyzes a system of one degree of freedom under the influence of a discontinuous (on-off) controller. The entire control system is assumed to be idealized to the extent that time lags, backlash, and other such unstabilizing influences are neglected. For this particular system, the analysis of motion is presented by means of the phase plane. Much of the paper is given to interpretation of the phase-plane curves in terms of what is physically occurring in the control system.

Not only are phase-plane diagrams presented, but also derived and graphed are loci of points of interest in the motion of the system, such as the conditions of reversal of the variables. Stability of the system is discussed, along with the possible courses of motion of the system, and the parameters useful for a desired course.

The paper is not easily read, perhaps because it is a translation of a German paper, and its way of expressing thoughts is at considerable variance with our own control literature. The fact that the symbols used are not clearly defined is a further impediment to understanding. A much more readable paper on the same subject, and one which covers most of the ideas expressed in the paper under review is "Analysis of relay servomechanisms," by H. K. Weiss [J. aero. Sci. 13, no. 7 (July 1946)].

William R. Ahrendt, USA

1034. Gérard Lehmann, Recent progress in the determination of precision and stability of servomechanisms (in French), Bull. Ass. tech. Marit. Aéro. 46, 79–110 (1947).

This expository paper is an excellent review of the application of negative-feedback amplifier theory to the behavior of servomechanisms. The Nyquist stability condition and the Bode gain vs. phase relations are utilized for illustrating the factors influencing the stability, precision and response of selected servos. Vincent Salmon, USA

Vibrations, Balancing

(**See also Revs. 1027, 1047, 1062, 1078, 1149, 1168**)

1035. H. Seifert, Periodic motions of mechanical systems (in German), Math. Z. 2, 197–216 (1948).

Let $U(x_1, \ldots, x_n) = U(x)$ be real and analytic, and let the set B of points which satisfy $U(x) = E$, E being some fixed real number, constitute a simple closed surface. Furthermore, it is supposed that $U(x) < E$ in the interior of B, and grad $U \neq 0$ on its boundary. Let $\Sigma a_{ij}(x)dx_i dx_j$ be a positive definite form with analytic coefficients. There is thereby defined a dynamical sys-

tem with Lagrangian coordinates $(x_1,\ldots,x_n)$, with potential energy U and kinetic energy $T(x,\dot{x}) = \Sigma a_{ij}(x)\dot{x}_i\dot{x}_j$. It is proved that this dynamical system possesses a periodic motion with total energy E, a point of this motion moving back and forth on an arc with end points on the boundary. Ed.

1036. I. G. Malkin, Oscillations of systems with one degree of freedom, close to Liapounoff systems; Oscillations of systems with several degrees of freedom, close to Liapounoff systems (in Russian), Prikl. Math. Mekh. 12: 561–596; 673–690 (1948).

The author observes that with the usual analytical methods which consider nonlinear equations as quasilinear (i.e., the coefficients of the nonlinear terms contain a small parameter), it is not possible in general to obtain the totality of periodic solutions. He studies systems

$$\dot{x} = -\lambda y + X(x,y) + \mu f(t,x,y,\mu),\ \dot{y} = \lambda x + Y(x,y) + \mu F(t,x,y,\mu) \quad (1)$$

where $X = -\partial H/\partial y$, $Y = \partial H/\partial x$ do not contain linear terms and f, F are periodic in t of period 2π; all these functions are supposed to be analytic. The nonlinear canonical system

$$\dot{x}_0 = -\lambda y_0 + X(x_0,y_0),\ \dot{y}_0 = \lambda y_0 + Y(x_0,y_0) \quad (2)$$

is called the generator system of (1). The application of classical methods to (2) yields the existence (provided λ satisfies certain restrictions) of analytic solutions $[x_0^{(n)}, y_0^{(n)}]$ periodic with period $2\pi/n$, n an integer. These generator solutions are the starting point for the calculation of periodic solutions of (1), analytic in μ:

$$x^{(n)} = x_0^{(n)}(t-\alpha) + \mu x_1(t) + \ldots,$$
$$y^{(n)} = y_0^{(n)}(t-\alpha) + \mu y_1(t) + \ldots \quad (3)$$

Application of Poincaré's methods gives the proof of the following theorem. For the existence of a periodic solution (3) of (1) it is necessary that α be a root, and sufficient that α be a simple root, of the equation

$$_0\int^{2\pi} [f_0 \cdot dy_0^{(n)}(t-\alpha)/dt - F_0 \cdot dx_0^{(n)}(t-\alpha)/dt]dt = 0,$$

where $f_0 = f[t,x_0^{(n)}(t-\alpha), y_0^{(n)}(t-\alpha),0]$, and F_0 is defined similarly. The practical calculation of the series follows the usual procedure of indeterminate coefficients. A similar theorem is proved for the existence of a solution (x^0,y^0) of (1) which tends to 0 as $\mu \to 0$.

The author considers next the "resonance" cases, where $\lambda = n + \mu a$; the previous method is not valid in this case. Assuming certain restrictions on the Fourier coefficients of $f(t,0,0,0)$ and $F(t,0,0,0)$, he is able to prove that a periodic solution (x_*,y_*) of (1) exists which tends to 0 as $\mu \to 0$ and which is analytic in $\nu = \mu^\omega$, $\omega = 1/(2s+1)$, s being an integer depending on the form of the solutions of (2).

After discussing stability questions, the author applies his methods to Duffing's equation:

$$\ddot{x} + k^2x - \gamma x^3 = \mu(a\cos pt + b\cos qt - 2h\dot{x}),$$

γ,μ,h positive; p, q integers. If q/p is not an odd integer, two real solutions $x^{(p)}$ exist when $p < k$. Several terms of the series developments of these solutions, as well as of x^0 and x_* (when $k^2 = p^2 - \mu\lambda$) are calculated, and the stability properties of these solutions are discussed in great detail with varying k.

Courtesy of Mathematical Reviews J. L. Massera, Uruguay

In the second paper the assumptions on the system are such that under a suitable real affine transformation of coordinates it takes the form

$$dx/dt = -\lambda y + X + f,\ dy/dt = \lambda x + Y + F,$$
$$dx_s/dt = \Sigma r_{si}x_i + X_s + f_s,\ s = 1,\ldots,m,$$

where all the functions are analytic in the coordinates near the origin, X, Y and X_s contain only the coordinates and these in powers not less than 2, while f, F and the f_s contain in addition μ and t. Moreover, the constant matrix $||r_{si}||$ has no pure complex or zero characteristic roots. Finally it is assumed that for $\mu = 0$ there is a general integral of the form $H = x^2 + y^2 + S(x,y,x_1,\ldots,x_m) =$ constant, where H is analytic at the origin and contains no linear terms, while the quadratic terms in S do not contain x or y. Under these assumptions the author succeeds in extending the results of his preceding paper to the more general situation. S. Lefschetz, USA

1037. E. Bright Wilson, Jr., Note on a form of the secular equation for molecular vibrations, J. chem. Phys. 15, 736–738 (Oct. 1947).

The solution of the problems of small-amplitude vibrations, as is well known, involves some type of determinantal or secular equation. By starting with the Hamiltonian form of the equations of motion in the problem of small-amplitude vibrations of the atoms of a polyatomic molecule, the author introduces a less-known form of a secular equation, twice the usual size. It is unsymmetric with respect to the principal diagonal, all elements of which are ω (with $\omega = 2\pi\nu$, where ν is the frequency of vibration). One off-diagonal corner is occupied by the elements of the force-constants matrix F, and the other corner by the inverse kinetic energy coefficients G_{ij}(G are the matrix elements). All other elements are zero. This new form of the secular equation is found to be particularly useful in molecular vibrational problems when a machine is used to solve the equation.

Eugene Leimanis, Canada

1038. T. Manacorda, An extension to second-order equations of a Hartmann-Wintner asymptotic formula for the number of zeros (in Italian), Boll. Un. Mat. Ital. 3, 205–210 (Dec. 1948).

An appraisal for $N(x)$, the number of zeros of a solution of $y'' + 2a(x)y' + \omega^2(x)y = 0$ over $(0,x)$, is obtained when there exists an l such that $0 \leq a(x) < l < \omega(x)$ and when $\omega'(x) = o(\omega^2(x))$. The appraisal yields an asymptotic relationship for the case $a(x) \equiv 0$ due to Hartmann and Wintner [Amer. J. Math. 70, 1–10 (1948)]. N. Levinson, USA

1039. Friedrich Wilhelm Schäfke, On the three pure coupling types of two vibrating systems (in German), Math. Nachr. 1, 66–80 (1948).

This paper relates to the theory of linear dynamical systems with two degrees of freedom, having systems of differential equations of motion of the form

$$x_1'' + 2e_1x_1' + k_1^2x_1 + \rho_1x_2'' + 2\sigma_1e_1x_2' + \tau_1k_1^2x_2 = 0,$$
$$x_2'' + 2e_2x_2' + k_2^2x_2 + \rho_2x_1'' + 2\sigma_2e_2x_1' + \tau_2k_2^2x_1 = 0,$$

the coefficients being real constants such that

$$\rho_2/\rho_1 = 2\sigma_2e_2/2\sigma_1e_1 = \tau_2k_2^2/\tau_1k_1^2 > 0.$$

It is assumed that the parameters $\rho^2 = \rho_1\rho_2$, $\sigma^2 = \sigma_1\sigma_2$, $\tau^2 = \tau_1\tau_2$ satisfy the relations $0 \leq \rho^2 < 1$, $0 \leq \sigma^2 < 1$, $0 \leq \tau^2 < 1$. The author studies the behavior of the roots of the characteristic equation

$$\begin{vmatrix} \lambda^2 + 2e_1\lambda + k_1^2 & \rho_1\lambda^2 + 2\sigma_1e_1\lambda + \tau_1k_1^2 \\ \lambda^2 + 2e_2\lambda + k_2^2 & \rho_2\lambda^2 + 2\sigma_2e_2\lambda + \tau_2k_2^2 \end{vmatrix} = 0,$$

when one of the parameters ρ,σ,τ is varied while the other two are

held fixed at the value zero. He also discusses some of the physical implications of the results of this study. The results given in the paper consist essentially of a large body of diverse details, and they do not admit of any brief summary.
Courtesy of Mathematical Reviews L. A. MacColl, USA

1040. Max Päsler, Resonance curves of forced vibrations due to perturbations with frequency dependent on amplitude (in German), Ann. Phys. Leipzig 4, 1-13 (Sept. 1948).

The vibration defined by $m\ddot{x} + 2\delta\dot{x} + cx = K_n\omega^n \cos \omega t$ is discussed. For $n = 0$ the resonance amplitude decreases, for $n = 1$ it remains constant, and for $n = 2$ increases with increasing c. The author denotes by σ_n the ratio of the resonant-vibration amplitude to K_n, and shows that the envelope of the "resonance curves" [$\log \sigma_n$, $\log \omega$] for variable c is a straight line of slope $n-1$. For $n = 1$ the resonance frequency coincides with the frequency $(c/m)^{1/2}$ of the undamped vibration; it is smaller for $n = 0$ and larger for $n = 2$. Heinz Parkus, Austria

1041. Fritz Schürer, On the theory of balancing (in German), Math. Nachr. 1, 295-331 (1948).

The equation governing regulated forced oscillations is $f''(t) = af(t) - hf'(t) - bf(t - \delta) - cf'(t - \delta) + g(t)$. The solution is a series of exponentials, obtained from a characteristic transcendental equation. Stable solutions result when all the characteristic roots have negative real part, and the author obtains simple conditions for this.
Courtesy of Mathematical Reviews P. Franklin, USA

1042. O. Föppl, Approximate graphical calculation of vibration frequencies due to torsion and bending (in German), Technik 2, 25-28 (Jan. 1947).

The problem consists of finding torsional and flexural frequencies of a weightless shaft carrying several masses, some of which may be outside the supports. A graphical Hölzer's method is shown to give the torsional frequencies. The frequencies of the first two bending modes are obtained by a graphical application of the conjugate-beam principle. For the second bending mode the procedure has the recommendation that there is no tendency to return to the fundamental. Criteria are established for determining when the construction gives, for practical purposes, the correct answer. The analysis is in symbolic form with no numerical examples. Robert P. Felgar, Jr., USA

1043. Fritz Sauter, On the theory of vibrations of thin elastic plates (in German), Z. Naturforsch. A 3a, 548-552 (Aug.-Nov. 1948).

The theory of thin elastic plates is developed from the general equations of motion for an elastic medium. Solutions of the general equations are sought in which displacements and stresses are of the form $a(z)e^{i\chi x - i\omega t}$, where x,y is the plane of the plate, having surfaces $z = \pm h$. Different types of solution involving shear, stretch or bending are obtained by taking appropriate displacement components to be zero. $a(z)$ is chosen to give an arbitrary number, n, of nodes through the plate thickness. These conditions determine frequency spectra for the wave velocity ω/χ as a function of the frequency ω, for the various types of motion and values of $n = 0, 1, 2, \ldots$. The partial differential equation for the motion is deduced from the solution $a(z)e^{i\chi x - i\omega t}$ and the relation between χ and ω implied by a particular spectrum. The usual thin-plate equations for shearing, extensional and flexural motion are deduced by taking the limit $h \to 0$. Better approximations including higher powers of h can also be deduced. For large h the appropriate spectrum gives the velocity of Rayleigh waves. E. H. Lee, USA

1044. Sven T. A. Ödman, Differential equation for calculation of vibrations produced in load-bearing structures by moving loads, Int. Assn. Bridge struct. Engng. third Congress, prelim. publ., 669-680 (1948).

The author considers the case of a single moving load whose mass cannot be neglected. The deformation at any instant can be expressed as

$$W(s) = \sum_1^\infty q_n(t)\phi(t,s) \tag{1}$$

where the characteristic functions ϕ_n vary with time, because the distribution of the masses in the compound system varies with the position s of the load. Also the natural frequency $\omega_n(s)$ of the compound system varies with s. The author does not neglect this variation and therefore his theory is more general than the usual one where one puts $\phi_n = (2/l)^{1/2} \sin n\pi x/l$.

If we insert equation (1) in Lagrange's equation of motion, we obtain an infinite number of differential equations in the generalized coordinates $q_n(t)$: we may disregard the influence of all vibrations except the nth. Hence the nth equation contains only the coordinate q_n. It is a differential equation of the second order, inhomogeneous and linear, with variable coefficients which contain the functions of the load's position: $\phi_n(s)$, $\dot{\phi}_n$, $\ddot{\phi}_n$; $\omega_n(s)$.

The author considers first the corresponding homogeneous equation of which he offers a solution

$$q_n = e^{-\int \beta_n(t)dt}\left(A \sin \int \bar{\omega}_n(t)dt + B \cos \int \bar{\omega}_n(t)dt\right) \tag{2}$$

where the damping β_n and the modulated frequency $\bar{\omega}_n$ are not constant but vary with time and are bound by two differential relations which must hold in order that the expression (2) satisfy the differential equation in q_n. Using some approximations, the author gives the solution of the inhomogeneous equation for q_n. The characteristic functions can be imagined as composed of trigonometric and hyperbolic functions in which the arguments are also variable with time. The author intends to continue the work to make a comparison with experimental results possible.

The theory applies to the case of several masses moving on the structure with constant velocities, and to arbitrary transverse forces. D. Gentiloni Silverj, Italy

Wave Motion, Impact

(See also Revs. 1186, 1193, 1194)

1045. M. Pastori, Wave propagation in anisotropic media and the correspondent principal directions (in Italian, with English summary), Nuovo Cim. 6, no. 3, 187-193 (May 1949).

The author shows that, in an anisotropic medium, there exists at every point at least three directions such that the discontinuity surfaces normal to these directions propagate at speeds whose sum of squares is constant in the medium. In an elastic Green medium these directions coincide with the classical principal directions. Tensor symbolics is used. Ed.

1046. H. Koppe, On Rayleigh waves at the interface of two bodies (in German, with Russian summary), Z. angew. Math. Mech. 28, no. 11/12, 355-360 (1948).

The author applies the Rayleigh wave theory to boundary waves between two media. The first case he considers is that of two semi-infinite isotropic elastic solids in welded contact. The irrational equation derived for the velocity of boundary waves between the two solids in terms of the elastic constants and the transverse wave velocities in the two media, is obtained on the assumption that a displacement potential exists for motions parallel to the interface, and that amplitudes normal to the inter-

face fall off exponentially with distance from the interface in both media. Solving numerically for the case of equality of the two Lamé constants, the author finds that boundary waves exist only if the ratio of the rigidity in the acoustically less dense medium to that in the denser medium lies between 3 and 0, and the boundary wave velocity lies between the velocity of Rayleigh waves and transverse waves in the denser medium. The second case the author considers is that of waves of a fluid in contact with a solid. He concludes that boundary waves can always exist and that they travel more slowly than the corresponding Rayleigh waves. [There are a few minor typographical errors, such as subscript z in place of X in the last term of Equation (2) on page 356, and $p = \frac{1}{3}$ instead of $q = \frac{1}{3}$ in line 6 on page 359.]

Courtesy of Mathematical Reviews J. B. Macelwane, USA

1047. **Henri Pailloux, On a bar drop** (in French), C. R. Acad. Sci. Paris 229, no. 22, 1118–1120 (1949).

A vertical uniform bar is supported on one end. The axial displacement is known at $t = 0$ when the support is suddenly removed. The subsequent longitudinal vibration satisfies the wave equation. The author solves this equation subject to the discontinuity of the initial conditions, and the final result is given in a trigonometric series. D. L. Holl, USA

Elasticity Theory

(See also Revs. 1045, 1067, 1070)

1048. **F. D. Murnaghan, The foundations of the theory of elasticity,** Proc. Symposia appl. Math. I, 158–174. American Mathematical Society, New York (1949).

The paper gives a general formulation of the theory of finite strain, valid under conditions such that plastic deformation does not occur (e.g., compression under hydrostatic pressure). Matrix notation is used, which makes the formulas very compact. Assuming the existence of a free energy function in isothermal change, the author obtains the following formula for the stress T resulting from a finite displacement which moves a particle from $a = (a^1,a^2,a^3)$ to $x = (x^1,x^2,x^3)$: $T = \rho(x)J^*(\partial\phi/\partial\epsilon)J$. Here J is the Jacobian matrix $\partial x/\partial a$, J^* its transpose, $\rho(x)$ the final density so that (by conservation of mass) $\rho(x) = \rho(a)|\det J|^{-1}$, ϕ is the free energy per unit mass, assumed a function of the strain $\epsilon = \frac{1}{2}(J^*J - E)$, E = unit matrix. Thus T is a function of the nine elements of $\partial x/\partial a$, but the author emphasizes that it is not, in general, a function of the strain ϵ only. The above formula for T appears to be the central point of the paper. From it specializations are made. Thus, for an isotropic body, ϕ is a function of the three invariants under rotation of the matrix ϵ. The case of hydrostatic pressure is considered, with $T = -pE$, $\epsilon = -eE$ (p,e scalars), and the following formula obtained: $p = A[(v_0/v)^a - 1]$, where v_0 is the value of the volume v when $p = 0$, and A, a are constants characterizing the medium. The author claims good agreement between this formula and the experimental results of Bridgman for the compression of indium. The uniform tension of an elastic cylinder is also discussed.

J. L. Synge, Ireland

1049. **Hans Richter, Distortion tensor, distortion deviator and stress tensor for finite deformations** (in German), Z. angew. Math. Mech. 29, 65–75 (Mar. 1949).

In the theory of finite strain it is usual to employ the deformation tensor which arises from consideration of the difference of the squares of line elements in the initial and strained states. The formation from this of the deviatorial tensor which characterizes change of form (in the absence of change of volume) encounters particular difficulties. These difficulties may be traced to the fact that an infinitesimal-strain tensor may be decomposed into the sum of a small stretch and a small rotation, whereas, for finite strain, decomposition is only possible into a tensor product of a stretch and a finite rotation and the multiplication is not commutative. The author makes a fresh start using matrix notation which involves matrices for the coordinate transformations, but shorn of this scaffolding, his main results are as follows: Strain can be described by the affine mapping of a vector element dr in the first state on a vector element ds of the second state by a tensor A, so that $ds = A \cdot dr$.

The tensor A can be written in the form $A = S \cdot R$, where S is a symmetric tensor which operates a pure stretch and R is a tensor which operates a pure rotation. Then if A_c, R_c are the tensors conjugate to A and R,

$$A \cdot A_c = S \cdot R \cdot R_c \cdot S = S \cdot S,$$

since $R \cdot R_c = I$, the identity tensor. The deformation tensor is then taken to be

$$L = \log S = \tfrac{1}{2} \log A \cdot A_c$$

This tensor has the additive superposition property for coaxial stretches S_1, S_2 (i.e., such that $S_1 \cdot S_2 = S_2 \cdot S_1$), for $\log S_1 + \log S_2 = \log (S_1 \cdot S_2)$. Moreover it agrees with the ordinary deformation tensor in the case of infinitesimal strain. Further, the deviatorial tensor can now be formed in the same way for finite as for infinitesimal strain. L. M. Milne-Thomson, England

1050. **André Herpin, Extension of Cauchy relations to the coefficients of elasticity of third order** (in French), C. R. Acad. Sci. Paris 229, no. 19, 921–922 (1949).

The author calculates the coefficients of the third-order terms in the equation for the potential energy of an isotropic body. For the case of a body satisfying Cauchy's relations, according to experimental results, the numerical values of coefficients obtained are approximately in accordance with the values indicated by L. Brillouin. Zd. Bažant, Czechoslovakia

1051. **Wilfred Kaplan, Numerical methods in the solution of problems of nonlinear elasticity,** Proc. Symposia appl. Math. I, 194–196. American Mathematical Society, New York (1949).

A discussion is given of the methods of relaxation and gradients for determining the equilibrium position of a body from the solution of a set of simultaneous nonlinear equations. A compromise method is suggested for such problems. No discussion is given of the convergence of the solution in those nonlinear cases, where the equilibrium position is unstable. S. Levy, USA

1052. **G. Sonntag, Bending and twisting moments of the half-space** (in German), Z. angew. Math. Mech. 28, no. 9, 263–270 (Sept. 1948).

The author determines the stress distribution in an elastic solid occupying a half space and subject to a concentrated couple applied at a point of the plane boundary. The moment of the couple is either in this boundary or normal to it. The solution is derived from the corresponding problems for a load consisting of a single force. Henry Favre, Switzerland

1053. **Ya. K. Il'in, Determination of stresses in a circular disk rotating about an eccentric axis** (in Russian), Doklady Akad. Nauk SSSR 67, 803–806 (1949).

This note contains an outline of solution of the following problem: Determine the stresses in a thin elastic circular disk rotating with constant angular velocity about the axis normal to the plane of the disk, at a distance δ from its center. The problem is re-

duced to the solution of a plane problem of elasticity in the manner of Muschelishvili. The solution deduced by the author specializes to known results when $\delta = 0$.

I. S. Sokolnikoff, USA

1054. Ewen M'Ewen, Stresses in elastic cylinders in contact along a generatrix, Phil. Mag. 40, 454–459 (April 1949).

The problem of determining the contact stresses in elastic cylinders in contact along a generatrix Ox has been solved by various scientists as a special case of Hertz's classic problem. However these solutions do not take into account the effect of a tangential frictional component of load over the contact area, which is of importance in practical cases. This paper is the first to give a solution for the case of tangential friction. The solution is valid only when the materials of the two cylinders involved have the same elastic properties.

In this case, for the stress at any point, the author reaches the following expressions:

$$\widehat{xx} = + \frac{4F\sigma}{\pi b^2}\,[m - z + \mu(y - n)],$$

$$\widehat{yy} = - \frac{2F}{\pi b^2}\left[m - 2z + 2\mu(y - n) + m\,\frac{z^2 + n^2}{m^2 + n^2} + \mu n\,\frac{z^2 - m^2}{m^2 + n^2}\right],$$

$$\widehat{zz} = - \frac{2F}{\pi b^2}\left[m - m\,\frac{z^2 + n^2}{m^2 + n^2} - \mu n\,\frac{z^2 - m^2}{m^2 + n^2}\right],$$

$$\widehat{yz} = - \frac{2F}{\pi b^2}\left[\mu(m - 2z) - n\,\frac{z^2 - m^2}{m^2 + n^2} + \mu m\,\frac{z^2 + n^2}{m^2 + n^2}\right].$$

These, if $\mu = 0$, give the well-known equations for the case without friction. Here $2b$ is the width of contact zone, E Young's modulus, F load per unit length of line of contact, μ coefficient of friction, σ Poisson's ratio, $A = b^2 - y^2 + z^2$; $m^2, n^2 = \frac{1}{2}[\pm A + (A + 4y^2z^2)^{1/2}]$.

Morris A. Bricas, Greece

Experimental Stress Analysis

1055. E. W. Kammer and T. E. Pardue, Electric resistance changes of fine wires during elastic and plastic strains, Proc. Soc. exp. Stress Anal. 7, no. 1, 7–20 (1949).

Static stress-strain and resistance-strain relationship for single wires of approximately one mil diameter were obtained experimentally for iron, nickel, platinum and 15 commercially available alloys, up to a strain of about 1 %. The applicability of each material for strain-gage use is discussed. Of the materials tested, it appears that those incorporating the best characteristics are in use in commercially available strain gages. In view of the current interest in plastic-strain measurement, the results of this paper may serve as a good guide in future investigations.

George Gerard, USA

1056. Dean Christian, BL–310 "strain analyzer," Proc. Soc. exp. Stress Anal. 7, no. 1, 21–29 (1949).

The paper describes equipment, manufactured by the Brush Development Company, for recording signals from electrical-resistance strain gages on a direct inking oscillograph. The unit is compact, stable, easy to operate, and capable of recording signals from static strains to strains varying up to about 120 cps.

Horace J. Grover, USA

1057. K. Verlaan, Condenser-strain gages (in Dutch), Ingenieur's-Gravenhage 61, no. 47, 0.89–0.93 (1947).

The ratio of two capacitances constituted by a set of three condenser plates, the middle plate of which is movable with respect to the mutually fixed outer ones, is measured by means of a Wheatstone bridge. It is evaluated in terms of the relative displacement of the middle plate, which accompanies the strain to be determined. The parallelism of the said plates is maintained by the use of a one-piece frame provided with a system of four elastic hinges at the vertexes of a parallelogram. Constructions with small or large measuring length are described, and a construction able to measure the extensional and shear strains at the same time is mentioned.

J. A. Haringx, Holland

Rods, Beams, Shafts, Springs, Cables, etc.

(See also Revs. 1077, 1082, 1096, 1097, 1103, 1105)

1058. F. Stuessi, The design of concrete beams of rectangular cross section with simple reinforcement (in Spanish), Informes Constr. 2, no. 15, article 422–6, 6 pp. (Nov. 1949).

This is a Spanish translation of a paper published in vol. 1 of Proc. Int. Assoc. Bridges Structures. The author analyzes the stresses in a reinforced-concrete beam at the moment of rupture, taking into consideration the plastic state of the steel. The author thinks that the concept of moment at rupture should be used in design instead of the more common assumption of elastic behavior.

Ed.

1059. A. G. H. Dietz, Two-species laminated beams, Trans. Amer. Soc. mech. Engrs. 71, 401–405 (May 1949).

This paper gives the theory of two-species wooden beams incorporating high-strength outer laminations with low-strength cores. The theory indicates that it is possible to make two-species beams using relatively large amounts of low-grade materials that will be as strong and stiff as solid beams of the high-strength materials. Experimental data are given that agree with the theory.

R. M. Wingren, USA

1060. J. Barthélemy, On the deformation and internal stresses of tubes (in French), Bull. Ass. tech. Marit. Aéro. 46, 411–463 (1947).

This paper gives coefficients and functions for use in the determination of stresses and deformations in plane curved tubes subjected to internal pressures as well as to external tractions. There is no methodical treatment, and the complicated results are based on previous papers by M. Thuloup on curved tubes [same source, 32, 36, 41 (1928, 1932, 1937, respectively)]. Theoretically determined stresses and deformations are compared with the results of tests by M. Leiris [same source, 37 (1933)].

While very complicated sets of coefficients and functions are developed for the purpose of calculating stresses, the equation which gives the flexibility of the tube is not essentially different from the simple one of von Kármán in the range of moderately thick tubes.

Curves of comparison between theoretical calculations and the experimentally determined stresses of Leiris are given. Also, very extensive and complicated formulas are given for the parameters which define the problem.

The approach to the problem is markedly more complex than those of Marbec [Mém. Génie Maritime (1905) and Bull. Ass. tech. Marit. Aéro. (1911)], Le Besnerais and Th. von Kármán [Z. Ver. deutsch. Ing. (1889, 1911)]. However, it must be emphasized that the treatment allows for the important effects of

internal pressure and initial out-of-roundness of the cross section of the tube. [See following review.]

W. H. Hoppmann, II, USA

1061. **H. De Leiris and J. Barthélemy, Determination of deformations and stresses in a pipe of oval section** (in French), Bull. Ass. tech. Marit. Aéro. 47, 147–165 (1948).

This paper presents the results of an experimental investigation to test the theory devised by the second author for the determination of stresses and deformations of a plane curved tube subjected to internal pressures as well as over-all tractive forces.

A steel pipe of 164-mm OD and 7-mm wall thickness was used. It was shaped so that its centerline formed a circle having a 785-mm radius. The oval shape of the final cross section of the tube is given approximately by the expression:

$$Z = r(1 + 0.1 \cos 2\psi)$$

where Z is the radius of the section for any positional angle ψ. The quantity r is a constant. Strains were measured with Huggenberger extensometers and displacements with special pneumatic extensometer.

Internal pressures were varied from 1 Kg/mm to 17 Kg/mm, and traction forces from 0 Kg to 800 Kg were applied at a distance of 1570 mm from the center of the test section. Various combinations of external traction and internal pressure were investigated. Measured stresses are plotted along with theoretical curves in eight charts. Both transverse and longitudinal stresses are given. The experimentally determined stresses are also given in two tables along with over-all displacements which measure the flexibility of the whole tube. Very good agreement exists between the measured and the theoretically determined stresses and deformations.

W. H. Hoppmann, II, USA

1062. **K. Ludwig, Transverse vibrations of a helical spring** (in German), Z. angew. Mat. Phys. 27, 29–31 (Apr. 1947).

The author undertakes to correct a paper by J. Dick [The transverse vibrations of a helical spring with pinned ends and no axial load, Philos. Mag. 33, 513–519 (1942)] by apparently assuming that the shearing forces on a deflected beam produce a rotation of the elements of the beam equal to the slope of the deflection curve due to these forces.

This assumption is, to this reviewer's belief, incorrect because the rotation due to shearing force of an element of a beam in deflection is small of higher order compared with the slope of the deflection curve [see Timoshenko, *Vibration problems in engineering*, §41, New York, 1928]. The author offers no reason for his assumption.

Heinz Parkus, Austria

1063. **J. A. Haringx, Elastic stability of helical springs at a compression larger than original length,** Appl. sci. Res. Sec. A A1, no. 5–6, 417–434 (1949).

In general, a helical spring cannot be compressed by more than its free length because of interference between coils; i.e., the ratio ξ between critical compression and free length is normally less than unity. The author shows, however, that by taking a spring of large index, a compression greater than the free length (ξ greater than unity) may be realized experimentally by starting at one end and slipping the slightly expanded coils over each other, one at a time, i.e., turning the spring inside out, after which the spring is extended. It is found theoretically that for ξ greater than unity but less than a certain value which depends on the ratio l_0/D and the method of fastening the ends, the spring is unstable (l_0 free length, D mean coil diameter). This instability is manifested experimentally by a toppling over and sliding of the coils on each other. Cases treated are (a) hinged ends, (b) ends free to move laterally but not free to rotate, and (c) clamped ends. It is found that for cases (b) and (c) no instability occurs below certain values of l_0/D regardless of the value of ξ. In making the analysis the criterion is used that the work done by external forces or moments applied to the spring while under compression must be positive for stability. The theoretical results are checked qualitatively by experiment, and deviations between experimental and theoretical results are explained on the basis of inaccuracy in the assumption of small angles of rotation of the cross section.

A. M. Wahl, USA

1064. **W. Wuest, The calculation of rhomboidal springs for the measurement of tensile forces** (in German), Feinwerk Tech. 53, 21–24 (Apr. 1949).

For the construction of tensile dynamometers sometimes rhomboidal springs are employed. Two parallel springs are used for one unit, the deformation of the springs being a measure of the force applied. This article deals with springs of a fish-belly shape. They consist of a flat spring, composed of circular sections with the same or different curvatures or having the form of a sine wave. The stress distribution and deformations of these springs are calculated. A discussion concerning the most economical use of the spring steel for a certain application is included.

R. G. Boiten, Holland

1065. **B. Mackenzie, Automobile suspension springs,** Proc. Inst. mech. Eng. (Auto. Div.), part III, 122–135 (1947–48).

This paper examines the present state of development of automobile springs, with particular reference to leaf, torsion bar and helical-coil springs. Their reliability must be attained at the minimum cost, and these two conflicting requirements necessitate some degree of compromise in their solution.

First, the suspension characteristics and efficiency of the different types of springs are briefly treated. Then, the three types referred to above are successively studied from a practical point of view; the following topics are considered:

(1) *Leaf springs*: spring proportions and stresses; spring liners and gaiters; leaf sections; leaf ends; shot peening and presetting applied to leaf springs; leaf springs materials and treatments.

(2) *Torsion bar springs*: description of some recent suspension designs; stresses induced in round torsion bars by presetting; design and manufacture.

(3) *Helical-coil springs*: general properties; natural oscillations; improvement by presetting and shot peening.

The paper ends with some considerations about fatigue failures, which are illustrated by remarkable photographs. It is followed by a detailed discussion contributed by many automobile specialists, and the author's extensive reply.

C. Massonnet, Belgium

1066. **R. M. Macarthur, Tooth contact conditions in spur and helical gears,** Engineering 168, no. 4377, 654–657 (1949).

The author discusses some of the possible variations in design and their influence on the stress conditions in gear teeth. The discussion deals principally with the effects of contact or surface-stress conditions on the proportioning of spur and helical gears. Little consideration is given to the effects of stresses due to bending of the teeth.

The analytical work dealing with stress conditions is based essentially on the equations obtained by Hertz. A discussion, however, is presented of deviations which may exist in conditions of tooth contact from the conditions assumed by Hertz.

The author also presents a series of curves summarizing various relationships of interest in the design of gears. By means of these

curves, the author indicates how an improvement in tooth-contact conditions can be frequently made. Several criteria are discussed for determining the amount of addendum modification which can be usefully applied.

Frank Baron, USA

Plates, Disks, Shells, Membranes

(See also Revs. 1053, 1074, 1077, 1078)

1067. Samuel Levy, Large deflection theory for rectangular plates, Proc. Symposia appl. Math. 1, 197–210. American Mathematical Society, New York (1949).

The author presents a complete and clearly written survey of work by himself and collaborators on the subject of finite transverse deflections of thin elastic plates. To be solved are two well-known simultaneous nonlinear partial differential equations subject to appropriate boundary conditions. The author's approach to these problems is through the use of trigonometric double-series expansions for Airy's stress function F and the transverse deflection W. These series satisfy the boundary conditions of simply supported edges. The case of built-in edges is reduced to the foregoing by introducing into the theory certain specific forms of surface loads which are equivalent to applied edge moments. The author points out that, while this procedure seems to work well, it would be good to have a proof that his use of divergent series is legitimate (as it undoubtedly is). The trigonometric series reduce the differential equations of the problem to an infinite system of simultaneous cubic equations for the coefficients of the series. This system is solved by setting all coefficients except the first few equal to zero and by solving the resultant finite nonlinear system by iteration. It is observed that more terms have to be taken in the linear part of the equations than in the nonlinear part, for satisfactory accuracy. Explicit results thus obtained for various conditions of loading and support are presented in a series of diagrams. The paper concludes with specific recommendations for additional investigations. To the list of references given in this paper, one could add papers by G. Schadel, K. Marguerre, C. T. Wang, and others who obtained numerical results, similar to a few of those here, using different methods of solution of the same system of differential equations.

E. Reissner, USA

1068. K. O. Friedrichs, The edge effect in bending and buckling with large deflections, Proc. Symposia appl. Math. 1, 188–193. American Mathematical Society, New York (1949).

The author presents a concise survey of a number of boundary-layer theory problems in linear and nonlinear elasticity. Among the problems which are discussed by means of their relevant differential equations are those concerning (1) linear bending of a flat plate under initial tension, (2) nonlinear bending and buckling of circular plates, and (3) nonlinear membrane theory of spherical shells. The problem of linear bending of spherical shells, the effective width problem in the theory of buckling of flat plates, and the problem of the nonlinear buckling theory of spherical shells are also discussed. The author emphasizes the importance of the mathematical problems concerning appropriate boundary conditions which arise in connection with the reduction in order of a system of linear and especially nonlinear differential equations describing a given physical situation. Such reduction is often possible if the existence of a boundary layer is given or discovered.

E. Reissner, USA

1069. G. A. Grinberg and Ya. S. Uflyand, On the bending of a rectangular plate with a clamped boundary under an arbitrary load (in Russian), Prikl. Mat. Mekh. 13, 413–434 (1949).

The existing solutions of the problem of small deflections of a clamped rectangular plate under an arbitrary normal load lead to laborious calculations for the bending moments and shearing forces. This paper is concerned with the development of computationally more tractable formulas. In section 1 of the paper the boundary-value problem is reduced to the solution of an integral equation, which is solved approximately. In sections 2 and 3, the approximate solution deduced in section 1 is used to construct practically useful formulas for deflection, bending moments and shearing forces for several special types of loading.

I. S. Sokolnikoff, USA

1070. H. Neuber, General theory of shells, I (in German), Z. angew. Math. Mech. 29, no. 4, 97–108 (Apr. 1949).

A theory of thin shells of any shape and boundaries with variable thickness h and under arbitrary load is developed. After a survey of notations (absolute differential calculus is used throughout) and of general geometric principles, the geometry of the shell is discussed in terms of curvilinear coordinates u, v in the middle surface, and w normal to it. Special attention is given to covariant three-dimensional derivatives (space-derivatives) of the components of a space-tensor, and first and higher derivatives with respect to w of vectors and symmetric tensors are replaced by such space derivatives.

Consistently stress-quantities are reduced to space-tensors. After having developed the stress tensor S with respect to w, the author derives a set of relations between components of S and their space-derivatives and components of the external load on the outer surfaces $w = \pm h/2$ (surface-conditions). As the usual stress-quantities in shell-theory, i.e., resultant forces and moments on a cross section are not directly expressible as space-tensors, the equilibrium of an element of the shell is exactly defined, i.e., by equating to zero the moments of order n ($n = 0, 1 \ldots,$) of the divergence of S. Hence an approximation of any degree to the three-dimensional state of stress can be reached in principle. Actually the calculations are carried out for the degree of approximation defined by $n = 0$ and 1. Div$S$ is developed with respect to w, and unwanted terms are eliminated by means of the surface conditions and the vanishing moments. From S and its derivatives for $w = 0$ two other tensors L and M are derived, which in the special case of a plate are identical to the tensors of the resultant forces and moments. For these six unknowns three equations of equilibrium are found in which h does not appear explicitly.

A. van Wijngaarden, Holland

1071. M. B. Millenson and S. S. Manson, Determination of stresses in gas-turbine disks subjected to plastic flow and creep, Nat. adv. Comm. Aero. Rep. no. 906, 16 pp. (1948).

In a previous article by one of the authors (Rev. 2, 1375) a finite-difference method was presented for the calculation of elastic stresses in gas-turbine disks subject to a temperature gradient. In the present paper, this method is extended to cover the effects of both yielding and creep. This is done by including terms representing plastic flow and creep in the expressions for radial and tangential strains. For the plastic-flow terms, successive approximations are applied, based on the stress-strain diagrams of the material for the temperatures existing at various radii of the disk, using the shear-energy theory of strength as a basis. Where the disk is subject to a succession of loading and temperature conditions, the plastic flow increments occurring for each condition are added to give a total value. Creep increments are obtained for small time increments as functions of equivalent stress, the usual expressions for biaxial states of stress being applied, these creep increments are summed for successive time increments. The method is applied to an example of a welded-blade turbine disk in order to show qualitatively how the stress dis-

tribution changes with variations in speed and temperature, such as occur under operating conditions. A. M. Wahl, USA

1072. Siegfried Schwaigerer and Robert Kobitzsch, The design of gaskets and flanges (in German), Technik **2**, 425–430 (Oct. 1947); **2**, 489–493 (Nov. 1947).

The authors present rather elaborate formulas for calculating stresses in flanges. They resolve the stress into two parts, one due to the external bending moments, and the other caused by internal pressure in the vessel to which the flanges are attached. The authors treat these two stresses separately, considering that the stresses due to clamping moments are less dangerous than the stresses resulting from internal pressure in the vessel. The authors show quite elaborate diagrams for the flanges, based on the assumptions that flange neck and the flange proper are intersection of two dissimilar cylinders. They discuss somewhat the effect of stress concentrations, and show curves for use in the design of such flanges. They also treat various types of gaskets such as diamond shaped, multiple diamond shaped, lenticular, and round, and they show various cases of calculating the size of gasket to be used. They give a chart showing the load-carrying capacities for various sizes of the various shapes of gaskets.

R. G. Sturm, USA

Buckling Problems

(**See also Revs. 1063, 1068, 1083, 1184**)

1073. S. B. Batdorf, A simplified method of elastic-stability analysis for thin cylindrical shells, Nat. adv. Comm. Aero. Rep. no. 874, 25 pp. (1947).

L. H. Donnell derived in 1934 an eighth-order partial differential equation for the radial buckling displacement of a cylinder, applicable when the wall is thin and the number of circumferential waves is large enough. This equation allows the complete solution of the buckling problem, for a given type of loading, to be represented by one curve, in terms of two dimensionless parameters, one of which contains the critical stress (e.g., the shear stress in torsion) while the other contains only cylinder dimensions. In part I of the present paper the author has applied Donnell's equation to four problems, namely: cylinders under lateral pressure, axial compression, hydrostatic pressure (on closed cylinders), and torsion. In all these cases the end conditions were those of simple support as far as the radial displacement w is concerned; the author shows what restrictions on the other displacements are implied in his solution, which takes w as a trigonometric series satisfying the simple support condition term by term. The results obtained are plotted together with those obtained by other investigators using different equations, and in some cases with experimental results. The results from Donnell's equation agree well in general with other calculated curves, with some disagreement for very short or very long cylinders.

In part II of the paper it is shown that a modified form of Donnell's equation can be used to good advantage in solving the buckling problems of cylinders and cylindrical panels with clamped edges. The modified equation is obtained by multiplying Donnell's equation by the operator ∇^{-4}, defined by the equation $\nabla^4\nabla^{-4}f = 0$. This modification implies certain boundary conditions on the axial and tangential displacements, which are discussed. Solutions are given for a number of problems of cylinders and curved strips loaded in shear or with combined shear and axial stress. Trigonometric series in the radial deflection are assumed and Galerkin's method used to determine the coefficients. Results are compared with solutions obtained by other investigators, where these are available. The author gives arguments for believing that his solutions based on Donnell's equation (or its modified form) are sufficiently accurate for many purposes despite their inability to satisfy arbitrary boundary conditions except on the radial displacement. [See also Revs. **1**, **437–442**, **809**, **816**.]

P. S. Symonds, USA

1074. Folke Odqvist, On the effective width of reinforced plane plates, Roy. Swed. Air Board Rep. Transl. no. 5, 24 pp. (1948).

The influence of bending stiffness on the effective widths of plate beams is investigated. The case of a single beam of length l in the middle of an infinite plate strip is considered. The plate is assumed simply supported along the edges. A stiffening factor, i, is introduced according to the ratio, $i = I^1/I$. I indicates the moment of inertia of the beam section without plate, and I^1 is the apparent moment of inertia of the beam with plate. The quantity I^1 and the stiffening factor i have been computed for three different cases of load: cosine load pattern along the beam, concentrated load on the beam, and uniform distribution of load over the plate.

R. L. Bisplinghoff, USA

1075. K. H. Boller and C. B. Norris, Elastic stability of the facings of flat sandwich panels when subjected to combined edgewise stresses, For. Prod. Lab. Rep. no. 1802, 16 pp. (Feb. 1949).

The authors describe experiments in which six different designs of sandwich construction, comprising two varieties of cork boards in combination of varying thickness of 24S-T alclad aluminum sheet, were tested in compression and in shear. The test pieces were so proportioned that failure occurred by local instability on the faces; that is, the pieces were not slender enough to buckle due to a general instability, neither was the material loaded to its ultimate strength.

The shear tests were carried out in the equivalent of a pin-jointed frame loaded diagonally, shear strain being recorded by electric-resistance strain gages. In general the skins were able to withstand stresses in shear 25% in excess of those which caused failure in direct compression.

The authors conclude that failure in shear is due to the compressive stresses but that the presence of tensile forces at right angles enables increased stresses to be withstood. They suggest that the load carried by a panel subject to edgewise compression only may serve as a conservative guide to that supported by a panel under combined edgewise stress, the maximum compressive stress being taken as a basis.

F. T. Barwell, Great Britain

1076. Manuel Stein, J. Lyell Sanders, Jr., and Harold Crate, Critical stress of ring-stiffened cylinders in torsion, Nat. adv. Comm. Aero. tech. Note no. 1981, 17 pp. (1949).

The agreement between the computed and experimental results is quite satisfactory but the computations appear to be very long and laborious. References are not made to several papers which were published a few years before this work was done and which show simpler and more nearly complete solutions.

R. G. Sturm, USA

1077. Roger A. Anderson, Some preliminary information on buckling and ultimate strength of unstiffened compression skin obtained through bending and compression tests on rectangular cross section aluminum tubes, Flygtekn. Försöksanst. Medd. Rep. 27, 25 pp. (1949).

The experimental program described in this paper was carried out to compare the actual stresses in the compression skin of rectangular tubes at the onset of buckling with critical stresses that were calculated according to existing theories for flat plates with boundary restraints similar to those encountered in the walls

of the tubes. Rectangular tubes were used because the stability theory for such a tube is identical in principle to that for a multiweb beam, such as might be found in the wings of high-speed aircraft.

The tubes were tested in direct compression and in pure bending. The plastic buckling loads were taken to be those at which a marked strain deviation was indicated by electrical strain gages attached to the compression surfaces of the tubes. Various ratios of web thickness to skin thickness were obtained by machining away part of the side walls of the rectangular tubes.

The test results indicated that the buckling stress for such an idealized multiweb beam may be calculated with the same degree of accuracy as the critical stress in an ordinary flat plate in pure compression. This is only possible when the proper edge condition is taken into consideration and when appropriate corrections are made for the reduction in plate modulus at buckling beyond the elastic range. Evan A. Davis, USA

1078. G. Salet, Buckling; elastic deformations not proportional to the load; application to slightly oval cylinders; analogies between buckling and vibrations (in French), Bull. Assn. tech. Marit. Aéro. 46, 393–410 (1947).

This is an expository paper on some elementary aspects of the subjects mentioned. A formula is given for the increase in the stresses in a cylindrical pressure vessel due to a small departure from the true circular form. P. C. Dunne, England

1079. Elbridge Z. Stowell, A unified theory of plastic buckling of columns and plates, Nat. adv. Comm. Aero. Rep. no. 898, 11 pp. (1948).

The buckling of perfect flat plates compressed in one direction above the elastic limit is calculated. Several edge conditions are satisfied, including that of free unloaded edges. Ilyushin's treatment is followed except that it is assumed that no part of the plate is unloaded during buckling. This assumption is justified on the basis of Shanley's similar conclusions for a column, and the fact that if unloading is considered, the computed strengths are higher than those measured for actual plates (the possibility that this is due to initial defects is ignored). Results are expressed in terms of the tangent and secant moduli. L. H. Donnell, USA

Joints and Joining Methods

(See also Revs. 1086, 1105)

1080. F. Koenigsberger, Welding technology, London, Cleaver-Hume Press, Ltd., 1949, viii + 280 pp., 214 photos, 42 tables. Cloth, 9 × $5\frac{1}{2}$ in., 21 s. ($3).

The subject of welding technology has been carefully covered both from the academic and the engineering points of view. Since it has been written abroad, it naturally reflects the results of British practice and techniques. The author has referred to literature in the field of welding published in the United States. The book is profusely illustrated. The general topics include a general survey of welding processes and their applications; the fusion and force-welding processes; the welding of ferrous and nonferrous metals; equipment for oxyacetylene, arc and electric-resistance welding; flame cutting and gaging; distortion and residual stresses; weld strengths and design principles; safety precautions. The book is considered suitable for students studying for the welding examinations given in England.

Frederick K. Teichmann, USA

1081. E. C. Hartmann, Marshall Holt, and A. N. Zamboky, Static and fatigue tests of arc-welded aluminum alloy 61S-T plate, Welding Res. Supplement 12, no. 3, 129–138 (1947).

This paper deals with an experimental study of static and fatigue strengths of welded aluminum plates of 15 different joint designs. The load applied was tensile and the stress range varied from zero to the maximum. Diagrams and tables allow a clear comparison of the designs. The results show that of all the joints tested none is superior in over-all performance to the simple butt joints. The authors point out that heat of welding reduces the static strength approximately 50%, and that tests on welded specimens without actual joint lose from 42 to 58% fatigue strength. This last comparison, however, is made with standard specimens of smaller cross section. The reviewer wonders whether this fact may not introduce some error in the obtained fatigue-strength reduction. A. J. Durelli, USA

Structures

(See also Revs. 1058, 1073, 1186)

1082. I. Toth, On the analytical calculation of two-hinged circular arches (in French), Ossature métallique, 204–214 (Apr. 1949).

This paper is an application of the classical elastic theory of two-hinged arches. The general expression of the horizontal reaction is solved, for given load condition, in the particular case of a symmetric arch with circular axis and constant section in terms of the characteristic lengths of the arch. The load conditions considered are: (1) load uniformly distributed along the axis and along the horizontal projection of the arch; (2) single concentrated vertical or horizontal load with an arbitrary point of application on the arch. Tables and graphs are given for practical calculation. Giulio Ceradini, Italy

1083. A. Voellmy, Interdependence of compression, bending and buckling strength of concrete and reinforced concrete (in German), Schweiz. Bauztg. 67, no. 38, 536–541 (1949).

A relation between stress and deformation for concrete in compression is assumed and expressed in algebraic form. This relation is then used to derive analytically the relations between strength, eccentricity of loading, and slenderness ratio for plain and reinforced concrete columns. A few comparisons with results of tests are made. E. Hognestad, USA

1084. Herbert Schulze, Deflection of steel truss bridges put in place by the overhand method (in German), Bautechnik 26, 33–38 (Feb. 1949).

The equations developed in the elastic-load procedure for obtaining truss deflections are used to obtain panel-point-angle changes due to bar stresses and to play in joints in a truss. These angle changes are not applied to a conjugate beam as elastic loads, but a procedure is set up for obtaining the equation of the elastic line from them. This equation depends upon the moment of inertia of the truss being h^2F_0, where h is the depth of the truss and F_0 is the upper chord area. To offset the error introduced by this assumption, expressions are included to provide for the effect of the transverse shears in each panel. The resulting equations are long and cumbersome. They appear to offer no advantages over standard elastic-load procedures.

Additional expressions are obtained for the effect of deflection at one of the supports. A table of predicted and observed deflections is presented, and their agreement confirms the author's statement that the effect of shear deflections and joint play cannot be ignored when computing truss deflections. (Aeronautical engineers have appreciated this fact for some 25 years.)

The data in the tables are not sufficient to permit comparing the author's method with the standard elastic-load procedure, but it is

doubted that they offer any advantages in speed or accuracy.
Joseph S. Newell, USA

1085. F. Takabeya and I. Uchida, A practical method of calculation for continuous Vierendeel truss bridges on many supports (in French), Memo. Fac. Engng. Kyûshû Univ. 11, no. 3, 107–124 (1949).

Using essentially the slope-deflection method, the equations of equilibrium and geometric continuity for continuous Vierendeel truss bridges are developed. A method of tabulating the equations is given which greatly simplifies the work of setting up the problem. It is shown how the resulting set of simultaneous equations can be reduced to a form which can be solved readily by a successive approximation procedure. An appendix (written in English) summarizes the method and works out in detail two numerical examples. The analysis is limited to parallel chord trusses.
Dana Young, USA

1086. C. J. G. Jensen, Ship structural members—part 4, Shipbuilder 56, 307–310 (Apr. 1949).

The author presents the results of tests which are a continuation of a program originally laid down by the Welding Research Council. Static tests were conducted of flexural members consisting of plating reinforced with various types of longitudinal stiffeners as in the scantling of ships. Comparisons are made of the bending moments, stresses and deflections of members with "unattached," lugged or bracketed stiffeners. Various sizes of end brackets are considered. The comparisons are for riveted and welded construction.

The author concludes that where welded construction is adopted, some reduction in scantlings is justified.
Frank Baron, USA

Rheology (Plastic, Viscoplastic Flow)

(See also Revs. 1071, 1102, 1107, 1112)

1087. D. C. Drucker, Stress-strain relations for strain hardening materials: Discussion and proposed experiments, Proc. Symposia appl. Math. 1, 181–187. American Mathematical Society, New York (1949).

The difference between "deformation" and "flow" theories of plasticity is discussed. In the former the state of strain is determined completely by the state of stress; in the latter strain increments are determined by the stress and the stress increments, so that the total strain is a cumulative effect depending on the path of loading. It is suggested that tests involving a stress system which is rotated without change of magnitude would distinguish between the two types of theory most simply. Means of carrying out and interpreting such tests are discussed. E. H. Lee, USA

1088. C. Torre, The limiting stresses of bodies under static load (in German), Schweiz. Arch. 15: 116–121 (Apr. 1949); 145–158 (May 1949).

The author discusses limiting stress conditions of the type $F(\sigma_1,\sigma_2,\sigma_3) = 0$, where $\sigma_1,\sigma_2,\sigma_3$ are the principal stresses. His apparent object is to correlate these conditions for ideally plastic metals, for brittle materials, and for cohesionless materials. The author's term "limiting stress" refers to states of stress at a point such that, e.g., plastic flow is initiated locally. He discusses nonhomogeneous stress states on the basis of a hypothetical solution from the problem of a hollow sphere subjected to steadily increasing external pressure; this solution is also taken as basis for a discussion of stress-strain laws of ideally plastic metals for nonhomogeneous stress states.

The reviewer does not see that much light has been shed on the fundamental problems concerning limiting stress states or of stress-strain relations. Several other topics are also discussed, such as loading characteristics, some geometric relations in two-dimensional limiting stress surfaces, calculation of slip-line fields, and others.
P. S. Symonds, USA

1089. F. F. Vitman and N. A. Zlatin, Resistance to deformation of metals at velocities 10^{-6}–10^{2} m/sec. II (in Russian), Zh. tekh. Fiz. 19, 315–326 (Mar. 1949).

The resistance of metals to plastic deformation was determined experimentally as a function of speed of deformation and temperature by forcing a nondeforming cone at various speeds into lead and soft-steel specimens at various temperatures ranging from −180 C to +200 C for the lead, and −180 C to +20 C for the steel. These tests revealed the existence of three well-defined laws, each applicable to definite ranges of temperature and speed of deformation. The first law expressing the relative resistance H to plastic deformation ϵ is

$$H_k''/H_k' = (\dot\epsilon_2/\dot\epsilon_1)^{m(T-T_0)}$$

It applies to test temperatures above recrystallization and at speeds of deformation low enough to allow appreciable recrystallization to occur during the deformation. This law was shown to hold also for tests by Nádái and Manjoine (J. appl. Mech. 1941) on aluminum, copper and low-carbon steel. Values of the constants m and T_0 are tabulated by the authors.

The second law expresses the resistance to plastic deformation in the relative form $H_k''/H_k' = (\dot\epsilon_2/\dot\epsilon_1)^n$ or in the absolute form $H_k = M[\dot\epsilon \exp(N/T)]$, where M, N and n are constants. Here recrystallization is not permitted to occur, and the applicable speeds of isothermal deformation are determined by the speed of relaxation of the material. These speeds are greater than those for which the first law is applicable.

The third law is shown graphically and applies to still higher speeds of deformation. Here the deformation process is no longer isothermal but becomes adiabatic. The upper limiting speed of deformation for the third law is taken to be approximately 100 meters per sec. At greater speeds a fourth law is to be anticipated in which inertia forces of the deformed material would play an important part.

The authors consider their tests to show the impossibility of the existence of equivalent temperatures and deformation speeds in the wide range of their variation. Walter W. Soroka, USA

1090. Paul Bastien and Pierre Azou, Influence of low temperatures on strength and ductility of hydrogenated steels (in French), C. R. Acad. Sci. Paris 228, 1337–1339 (Apr. 1949).

The "true" fracture stress of normalized 0.15% C steel between +15 C and −70 C is reduced from about 100–110 kg/mm² to about 70–80 kg/mm² by electrolytic (48 hr) or nonelectrolytic etching in 10% HCl; the stress-strain curves show little change. The bottom of the fracture cup is bright and grainy; its relative area increases with decreasing temperature according to a curve consisting of three horizontal steps. These are attributed to the reduction of the number of operative slip planes from 3 to 2 at −20 C (at −10 C in the etched, +5 C in the electrolytically etched specimens), and from 2 to 1 at −70 C (at −50 C in the etched, −40 C in the electrolytically etched specimens).
E. Orowan, England

1091. Gunther Leibfried and H. D. Dietze, On the theory of the screw displacement (in German), Z. Phys. 126, no. 10/12, 790–808 (1949).

Both elasticity theory and the atomic point of view of Peierls

are employed to study screw dislocations, $w = (\lambda/2\pi)$ arc tan y/x in an infinite body. Here w is the only component of displacement (z direction) and λ is the atomic spacing. Solutions are obtained for stress, energy, and velocity of propagation of dislocations in an infinite medium, and also for the plate of finite thickness. When the dislocation is stationary at $x = 0$, $y = \eta$ in a plate extending between $y = 0$ and $y = D$,

$$w(x,y) = \frac{\lambda}{2\pi}\left\{\text{arc tan}\,\frac{\tanh(\pi x/2D)}{\tan \pi(y - \eta/2D)} - \text{arc tan}\,\frac{\tanh(\pi x/2D)}{\tan \pi(y + \eta/2D)}\right\}.$$

D. C. Drucker, USA

1092. V. V. Sokolovskiĭ, Approximate integration of the equations of a plane problem of the theory of plasticity (in Russian), Prikl. Mat. Mekh. 13, 321–322 (1949).

The author shows that under certain conditions the basic equations of two-dimensional plasticity can be written approximately so as to permit integration in closed form. This approximation is based on the replacement of exp $(\xi + \eta)$ by an expression of the form $[A(\xi + \eta + a)]^{2n}$, where ξ, η are the characteristic parameters and A, a, η are constants. Obviously, this is possible only if the range of $\xi + \eta$ happens to be sufficiently small for the problem under consideration. W. Prager, USA

1093. V. V. Sokolovskiĭ, On a plane problem of the theory of plasticity (in Russian), Prikl. Mat. Mekh. 13, 391–400 (1949).

The paper is concerned with states of plane strain in a plastic medium which obeys a yield condition of the type proposed by O. Mohr. If σ_1 and $\sigma_2 \leq \sigma_1$ denote the principal stresses in the plane of strain, the yield condition is assumed to have the form

$$(\sigma_1 - \sigma_2)/2k = \sin[(\sigma_1 + \sigma_2)/2k] + H/k,$$

where H and k are constants and $\sigma_1 + \sigma_2$ is assumed to satisfy $-2H \leq \sigma_1 + \sigma_2 \leq -2H + k\pi$. W. Prager, USA

1094. P. S. Symonds, On the general equations of problems of axial symmetry in the theory of plasticity, Quart. appl. Math. 6, 448–452 (1949).

It has been previously found that axially symmetric plasticity problems, in which the yield condition is taken to be that for which two principal stress differences equal the yield stress in pure tension, reduce to statically determinate problems. The method of characteristics can then be employed. The present paper investigates the question of whether the same problems with the von Mises flow condition lead to hyperbolic differential equations. The equilibrium conditions, yield condition, stress-strain laws (of the Saint Venant, Lévy, von Mises flow theory), and the compatibility equations are set up in terms of the stress-deviations and displacements in cylindrical coordinates. From these, the equations of the characteristic curves can be determined. It is found that the characteristics are real if and only if the problem is one of plane strain. Thus the equations are hyperbolic if and only if the axially symmetric case reduces to plane strain. G. H. Handelman, USA

1095. K. N. Shevchenko, An approximate solution in a closed form of a plane elastoplastic problem with axial symmetry (in Russian), Prikl. Mat. Mekh. 13, 323–328 (1949).

Using the stress-strain relations of Hencky, the author gives an approximate analysis of the stresses and strains in a thick-walled elastoplastic cylinder under interior and exterior pressure. [Since these stress-strain relations are of the deformation type, their use is justified only if the general conditions for the validity of stress-strain relations of this type are satisfied. The author does not offer any proof that this is the case.]

W. Prager, USA

1096. L. M. Kachanov, On the theory of transient creep (in Russian), Prikl. Mat. Mekh. 13, 381–390 (1949).

Creep is treated as a process of nonlinear viscous flow. The viscous strain rate ϵ_{ij} is supposed to be related to the stress σ_{ij} and the stress rate τ_{ij} by means of

$$\epsilon_{ij} = B(t)T^{m-1}(\sigma_{ij} - {}^1\!/_3\,\sigma_{kk}\delta_{ij}) + (1/2G)[\tau_{ij} - (\nu/1+\nu)\tau_{kk}\delta_{ij}],$$

where δ_{ij} is the Kronecker delta, T a quadratic invariant of the stress deviation $\sigma_{ij} - \sigma_{kk}\delta_{ij}/3$, m a constant, and $B(t)$ a monotonically decreasing function of time. A variational principle for materials of this type is established, and various examples are discussed, including flexure and torsion of bars and the general relaxation problem, where the displacements are prescribed on part of the surface of a body while the tractions are required to vanish on the remainder of the surface. W. Prager, USA

1097. L. A. Galin, The elastoplastic torsion of prismatic bars (in Russian), Prikl. Mat. Mekh. 13, no. 3, 285–296 (1949).

The paper constitutes a systematic study of V. V. Sokolovsky's inverse method of obtaining distributions of shearing stress which arise in the elastoplastic torsion of prismatic bars [Prikl. Mat. Mekh. 6, 241–246 (1942)]. It is assumed that the plastic region completely encloses the elastic region. In view of the yield condition, the components of shearing stress in the plastic region are then written in the form $\tau_x = k \sin \psi(t)$, $\tau_y = k \cos \psi(t)$, where k denotes the yield stress in simple shear and ψ is a function of the parameter t. The contour of the cross section and the elastoplastic stress distribution are represented in terms of the function $\psi(t)$. The solution of the direct problem is reduced to the solution of a nonlinear integral equation for $\psi(t)$. The following special cases of the inverse problem are discussed: $\psi(t) = t$ (circle) and piecewise constant $\psi(t)$ (nearly polygonal cross section). W. Prager, USA

Failure, Mechanics of Solid State

(See also Revs. 1065, 1077, 1089)

1098. W. Weibull, A statistical representation of fatigue failures in solids, Trans. roy. Inst. Technol. Stockh. 27, 51 pp. (1949).

The author's work on the statistical aspects of rupture phenomena began more than ten years ago and has had epoch-making significance in clarifying the thinking on the subject. The present publication is devoted to the subject of fatigue. The complete fatigue diagram represents relations between probability of failure p, load s, and number of cycles n. By applying sound statistical theory to the process of developing functions of these quantities from experimental data with the usual scatter, the author succeeds in gaining much insight into the phenomenological part of the fatigue problem. In particular, the extrapolation of fatigue data to larger numbers of cycles and the associated question of the existence of an endurance limit are much elucidated. The conclusion that aluminum does not have an endurance limit seems unavoidable.

One of the striking conclusions, which the author has published previously, is the size effect in fatigue. Two series of specimens of the same ball-bearing steel, but in which the specimens of one series were double the length of those of the other, are satisfactorily correlated.

No attempt is made to penetrate the physical nature of fatigue fractures. Sound phenomenological reasoning such as presented here is of utmost importance, however, and in an indirect sense it will have influence upon the clarification of the physical problem. The same methods are certain to be of significance in certain static rupture phenomena, such as rupture of materials at high temperature, where a phenomenological description, patterned on the fatigue problem, is possible. C. Richard Soderberg, USA

1099. **E. Siebel and K. Ruhl, Determination of the yield point for the strength of materials under tension** (in German), Technik 3, 218–223 (May 1948).

There is introduced an approximation method of calculating the yield points of materials subjected to different types of stress distribution. The method is based upon the assumption that, when the yield stress of the material is exceeded locally, the subsequent deformation will be proportional to that of the regions that are still elastic. If the stress-strain curve of the material is known, it is easy to calculate the local stress distribution in the region of plastic behavior. When the plastic distribution is known, the real yield point for the actual stress distribution can be related to the yield point for pure tension. It is obvious that the method might give a true result in the case of very small plastic deformations. Tests were made on two different materials, and the results show some agreement with the principal assumption.

Ragnar Nilson, Sweden

Design Factors, Meaning of Material Tests

(See also Rev. 1103)

1100. **H. Hagen, Criterion for suitability of materials to withstand static, dynamic, and thermal stresses based on the principle of similitude** (in German), Technik 3, 6–14 (Jan. 1498).

Utilizing considerations of the principle of similitude and related dimensionless quantities, the author provides a study of products, ratios, roots, and powers of physical constants which may serve as criteria for the suitability of materials of construction, especially under conditions of temperature change. He examines the derived dimensionless, as well as dimensional, quantities in some detail for the cases of static stress, body-force stress, dynamic (inertia) stress, and thermal stress. For thermal stress problems there is introduced a number, $c\gamma/E\beta$, in which $c\gamma$ is specific heat per unit volume, E is Young's modulus, and β is the coefficient of expansion. The author claims that this dimensionless quantity is a new one for studying similitude problems involving thermal stress. Its value is tabulated for a number of metals and ceramics. A chart then shows how ceramics in general differ from metals in terms of this quantity. There is also an interesting set of curves which show how the various suitability coefficients vary as functions of temperature from 0 C to 800 C. It is the opinion of the reviewer that these curves emphasize the difficulty of experiments with models for which temperature is one of the essential variables. W. H. Hoppmann, II, USA

Material Test Techniques

(See also Rev. 1064)

1101. **Toshio Nishihara, Shuji Taira, and Naoto Iwata, Investigation on the X-ray stress measurement of plastically deformed mild steel** (in Japanese), Trans. Soc. mech. Engrs. Japan 13, no. 45, 119–124 (1947).

It has hitherto been ascertained that stress values given by X-ray measurement are in good agreement with those measured mechanically in the elastic range so far as steel, cast iron, and light metals are concerned. Since, however, the agreement is doubtful beyond the yield point, the authors experimented on mild-steel specimens subjected to tensile stresses in plastic range, and found that the stress value by X-ray measurement is always lower than that measured mechanically. In order to inquire into the cause of this difference, the authors measured the distribution of residual stresses in the interior of plastically elongated specimen by the X-ray method, corroding it gradually from the surface, and found the state of residual stress of tension in the outer side, and of compression in the inner side. The same result is found in a plastically compressed mild-steel specimen. Thus the difference is ascribed to the state of nonuniform stress, due to plastic deformation, in the interior of the specimen.

Teruyoshi Udoguchi, Japan

1102. **Christian Boulanger, Contribution to the study of internal friction** (in French), Rev. Métall. 46: 255–265 (Apr. 1949); 321–342 (May 1949).

The paper presents a concise nine-page review of the scope of the literature on the subject to date. It is pointed out that wide differences exist in the data on internal friction reported by various researchers. Such differences are ascribed largely to absorption of energy by apparatus employed from the specimen. To overcome this a modified Coulomb micropendulum was employed, with this apparatus itself suspended from a torsion wire. Energy dissipation to the apparatus was avoided by use of a 0.1-gram specimen, with the apparatus weighing 25 kg. Dynamic torsional equations for this system are presented.

A brief critical discussion of various experimental techniques is presented. Results confirm present knowledge in the field. Moreover, the author believes that the present method of measurement is sufficiently exact to eliminate heretofore uncontrolled variables, and to show that internal friction is independent of specimen size and form.

In addition to the excellent discussion, a bibliography of 156 articles is included. M. E. Shank, USA

1103. **S. F. Dorey, Large-scale torsional fatigue testing of marine shafting,** Proc. Inst. mech. Engrs. 159, issue 46, 399–415 (1948).

The paper describes the design and construction of a new type of torsional fatigue-testing machine of the resonance type, capable of generating a reversed harmonic torque of up to $\pm 3 \times 10^6$ in.-lb in a stationary test shaft, at a frequency of approximately 2500 vibrations per min. Reference is made to a specially developed electronic method of speed control capable of regulating the nominal stress in the specimen to within 1 %.

The paper includes the results of a number of tests carried out on $9^3/_4$-in-diam mild-steel shafts, and also on Mechanite cast-iron specimens of 6 in. diam, and concludes with some remarks on scale effect, based on results from $^1/_2$-in-diam specimens subjected to reversed torsional fatigue tests carried out on a combined stress-fatigue-testing machine at the National Physical Laboratory.

From author's summary by K. W. Johansen, Denmark

Mechanical Properties of Specific Materials

(See also Revs. 1081, 1083, 1089, 1090, 1098, 1103)

1104. **Clarence M. Zener, Elasticity and anelasticity of metals,** University of Chicago Press, Chicago, 1948, vii, x + 170 pp., 56 figs. Cloth, 6 × 9 in., $4.

As stated in the foreword, the book deals with a subject on the borderline of metallurgy, physics and engineering. The term

anelasticity has been defined to denote that property of solids by virtue of which stress and strain are not single-valued functions of one another in that low-stress range in which no permanent set occurs, and in which the relation of stress to strain is still linear.

The book treats elasticity rather briefly and discusses only the topics of the coefficients relating stress to strain, and the dependence of these coefficients on the various physical parameters. The subjects of low-temperature elastic constants in cubic metals, the temperature dependence of elastic constants, and microelasticity are discussed in this section of the book.

The section dealing with anelasticity presents first the formal theory, including a generalization of the elastic equation, Boltzman superposition principle, and the relaxation spectrum. One chapter is devoted to the measurement of the relaxation spectrum, discussing the relation between measures of internal friction. The major part of the book is devoted to the physical interpretation of anelasticity, and discusses both homogeneous and inhomogeneous relaxation, treating thermodynamic variables, theory of relaxation by thermal and atomic diffusion, and relaxation of ordered and of preferential distributions induced by stress. The concept of the two-component systems, stress relaxation along previously formed slip bands, and stress relaxation across strain boundaries are also discussed.

In the opinion of the reviewer the book should do much to help the engineer realize that the fields of metallurgy and physics are turning up much information about the mechanical behavior of metals that is of interest to him and important in the solution of many of his problems. It is a book that should be of interest to all those concerned with the mechanical properties of metals.

C. O. Dohrenwend, USA

1105. Ambrose H. Stang and Bernard S. Jaffe, Tests of large welded-steel box girders, Weld. Res. Suppl. 14, 89–97 (Mar. 1949).

Bending tests are reported on four large welded-steel box girders, 30 in. wide, 25 in. deep, and with a span of 22 ft. Each test was run at one of the following temperatures: −40 F, 0 F, 40 F, 80 F. Failures were produced only in the beams tested at −40 F, and 0 F, after the center of the span had deflected 2.45 in. and 8.83 in. respectively. The tests at 40 F and 80 F produced no failure after deflections exceeding 16 in. Tensile tests of the material at these temperatures had indicated the steel was equally ductile at −40 F and 80 F.

Alexander Yorgiadis, USA

1106. P. H. Petersen, Burned shale and expanded slag concretes with and without air-entraining admixture, J. Amer. Concr. Inst. 20, no. 2, 165–175 (1948).

The physical properties of several lightweight aggregate Portland-cement concretes made with burned shale or expanded slag were investigated at the National Bureau of Standards. Three grades of concrete were made with each aggregate. Air entrainment greater than 20% is reported for the mixtures leanest in cement, an air-entraining admixture being used to increase the workability of all but the richest concretes. Compressive, flexural and bond-strength data are given, as well as resistance to heat transfer, rain penetration, and water penetration by capillarity. Also included are the coefficients of thermal expansion, shrinkage, and values for change in length due to wetting and drying.

Author's synopsis

1107. F. Guye, On the viscosity of cement paste (in French) Rev. Matér. Constr. no. 403, 117–122 (Apr. 1949).

An experiment is described in which it was attempted to measure the viscosity coefficient of cement-water pastes in a capillary-tube type viscometer, using the Hagen-Poiseuille law in the interpretation of the test results. As might have been expected, these results show considerable and consistent deviations from the straight-line $V(p)$ relation through the origin, characteristic of the simple Newtonian liquid. Introducing the Buckingham-Reiner viscometer equation for the viscous liquid with a yield limit (Bingham body), instead of the Hagen-Poiseuille equation, the test results can be adequately interpreted. The coefficient of viscosity of cement pastes is shown to decrease considerably with increasing water content and increasing fineness.

A. M. Freudenthal, USA

1108. Ralph W. Kluge, Morris M. Sparks, and Edward C. Tuma, Lightweight-aggregate concrete, J. Amer. Concr. Inst. 20, 625–642 (May 1949).

The paper reports a comprehensive series of tests at the National Bureau of Standards of concretes made with commercially available lightweight aggregates. The aggregate materials are exfoliated vermiculite, sintered diatomite, three expanded blast-furnace slags, sintered fly ash, pumice, expanded shale, expanded slate and expanded clay. Data are given for sieve analyses, specific gravity, absorption, unit weight, grading and crushing strength of the aggregates.

Four concrete mixes, ranging from 1:3 to 1:9 (approx.) cement-aggregate proportions by volume, were made with each aggregate. About 0.05% Vinsol N.V.X. by weight of cement was used to reduce segregation and bleeding in the wet mix. Complete data given for these 44 mixes include mix proportions; cement factor; slump and workability description; gal water per yd of concrete; unit weight; per-cent absorption; compressive and transverse strength; static and sonic modulus of elasticity; thermal conductivity; durability factor from freezing and thawing tests; and shrinkage at 100 days' age.

These tests provide valuable data for comparing or using the aggregates included in the study. The concretes made cover a range of weight from 24 to 119 lb per cu ft in room-dry condition, with 28-day compressive strengths from 95 to 7045 psi.

Henry A. Lepper, Jr., USA

1109. E. W. Scripture, Jr., and F. J. Litwinowicz, Effects of mixing time, size of batch and brand of cement on air entrainment, J. Amer. Concr. Inst. 20, 653–662 (May 1949).

This is a continuation of the paper of Rev. 3, **502**. In the present paper the influence of (a) mixing time and (b) size of batch upon the amount of entrained air is described. It was found (1) that, with or without an air-entraining agent, a maximum air content is attained in the early stages of mixing and thereafter decreases with further mixing; (2) as long as mixing is vigorous, the air content of the mix is not significantly affected by batch size.

M. Reiner, Israel

1110. Frederick F. Wangaard, The mechanical properties of wood, New York, John Wiley & Sons, 1950, xii + 377 pp., 88 figs. Cloth, 8.75 × 5.75 in., $6.

This book is an extensive up-to-date revision of Garratt's *Mechanical properties of wood* (1931).

Part 1 deals with stress-strain relations for wood. Much of the discussion in this section is elementary general mechanics of materials, although with special reference to wood. Detailed tables of the strength properties of all commercially important American woods, in both the unseasoned and air-dry conditions, are included.

Part 2 is concerned with factors affecting the mechanical properties of wood. These have been segregated into two main groups: defects, and factors other than defects. Among the factors covered in this section are the effect on the quality

of wood of growing conditions, position in tree, heartwood and sapwood, season of cutting, method of seasoning, and preservative treatment.

Part 3 is devoted to working stresses for structural lumber. Included are new tables of basic stresses as recently by the Forest Products Laboratory, and working stresses for stress-grade lumber as recommended by the National Lumber Manufacturers Association. Tables showing the relationship between defects of varying size or degree and strength ratios permit estimation of the weakening effect of knots, cross grain, checks, etc., upon any load-bearing member.

Part 4 describes currently recommended methods of timber testing. These are for the most part the methods used by the Forest Products Laboratory. In laboratories equipped with the necessary testing facilities, the instructions may be used as a manual in the conduction of mechanical tests.

An appendix contains a sample working plan for testing small clear specimens of wood. A list of references has been included at the end of each part of the book.

From author's summary by K. W. Johansen, Denmark

Mechanics of Forming and Cutting

1111. **Arthur R. Kimbell, Experimental determination of metal drawing and forming forces,** Proc. Soc. exp. Stress Anal. 7, no. 1, 51–60 (1949).

Load-travel diagrams are reported for six redrawing operations in an experimental press. The load is measured by resistance strain gages. The bearing of the results on die wear is discussed qualitatively.

Rodney Hill, England

1112. **T. Dahl, Present knowledge of warm deformation** (in German), Stahl Eisen 68, 333–345 (Sept. 9, 1948).

This paper describes the mechanism of plastic flow as the slip process inside the crystallite but not the slip of grains against each other. There are many theories concerning the flow strength, but the maximum-shearing-stress theory proved to be reasonable and practical. The effects of the friction and the shape of the compressive surface were discussed. Reduction in thickness by squeezing, forging and rolling are considered in the paper. Both the distributions of reduction in thickness transversely and stretching of the metal longitudinally are discussed. In the rolling process the authors consider the angle α_1 to be the rolling angle between the points of entry and exits on the roller. The condition for the rolls having a good grasp on the material to be rolled is that $\alpha_1 < \mu$, where μ is the angle of friction between rolls and the material to be rolled. When the material once is held between two rolls, the conditions for it to be pulled through is $\alpha_1 < 2\mu$. The coefficient of friction between the rolls and the material to be rolled can be determined experimentally by adjusting the rolling angle such that the rolls are just beginning to grasp the material.

In general, a rolling angle of 24 deg for smooth rolls and 34 deg for rough roller is allowable. If the rolling velocity is high, smaller rolling angles should be used. The dependence of the coefficient of friction on other variable factors such as temperature, roll surface, metal surface during rolling is of great importance since it really determines the possible amount which can be rolled down.

Finally the authors indicate that the greater the coefficient of friction between the rolls and the rolled material the higher the rolling resistance will be. They make the doubtful statement that the rolling resistance will be greater because of the greater contact between the rolls and the rolled material, but for a given reduction per pass it is not possible to see how such variations would exist. The authors point out that to reduce the flow resistance one should reduce the coefficient of friction, the diameter of the roll, or to apply the longitudinal pull to pull the rolled material through the mill.

S. W. Wang, USA

Hydraulics; Cavitation; Transport

(See also Rev. 1152)

1113. **T. Widell, Pressure losses in the flow of gas and liquid mixtures in horizontal pipes,** IngenVetenskAkad. Tidsk. Tekn. Forsk. 20, no. 2, 60–74 (1949).

This paper reports on measurements of pressure drop of flows of air and water mixtures, and a few measurements also for water vapor and water. Three tubes were used in the first series, having diameters of 25 and 36 mm (brass) and 84.8 mm (glass). For the steam tests a 25.6-mm copper tube was used. The results cover a range of liquid/gas-flow ratios, and are plotted and tabulated both as friction coefficient and as pressure-drop quotient $\Delta p/\Delta p_g$, where Δp is the pressure drop and Δp_g is the pressure drop that would occur if gas alone were flowing in the tube. The friction-coefficient curves do not exhibit a regular behavior with varying air speed and water content. These data are examined in the light of previous measurements of "critical speeds" of such mixtures [H. Edenholm (in Swedish), same source 3, 72–85 (1938)], and it is concluded that here the air and water are flowing separately. Computations and tests have been reported for this case by Martinelli et al. [Trans. Amer. Soc. mech. Engrs. 66, 139–151 (1944) and 70, 695–702 (1948)]. The present results are not in quantitative agreement with either theoretical or experimental results of the earlier paper, nor has any satisfactory theory been found by the author. When plotted in a certain way, the results for air and water coincide with those for steam and water.

W. R. Sears, USA

1114. **Hans Hartzell, Discharge coefficient μ in the formula for overfall dams** (in English), Acta Polyt. Stockholm 43, 3–25 (1949).

In this paper the author discusses the value of the coefficient μ in the formula $Q = \frac{2}{3}\mu(2g)^{1/2}BH^{3/2}$ for the discharge through a constricted section in a watercourse or over an overfall dam. The formula is derived from basic theory, and it is shown that, when hydrostatic pressure distribution in the stream above the dam crest is assumed, the theoretical value for μ is $(3\alpha)^{-1/2}$, where α is a coefficient expressing the energy loss owing to nonuniformity of velocity, contraction, gate supports, friction, etc., and has a value of about 1.1.

Experiments by Lindquist in Sweden on a model of a rectangular stop-log weir are discussed at length and compared with the formula. The experiments indicate a value for μ which is close to the theoretical value but varies with the H_0/L ratio, where H_0 is the height of the energy line over the weir crest, and L is the downstream length of the weir crest. This variation is caused by nonfulfillment of the basic assumptions, and has a trend to be expected from theory.

The formula can also be applied to a spillway with rounded crest if a pressure distribution other than hydrostatic is assumed. The coefficient μ becomes a function of H_0/R and α, where R is the radius of curvature of the spillway crest, and H_0 and α have the previous meaning. Comparison with experimental data again shows good but not exact agreement.

The paper is considered of great value to hydraulic engineers who are interested in fundamental theory, and who desire the best possible estimate of the discharge over dams and spillways.

Karl E. Schoenherr, USA

1115. Albert Schlag, Instruments for measuring of discharges: Discharge coefficients of Venturi unit normalized tube (in French), Hommage Faculty appl. Sci. Univ. Liége, 142–148 (1947).

The author compares discharge coefficients of normalized nozzles and Venturis. He analyzes various publications and gives detailed references. Limiting his comparisons to ranges where effects of the Reynolds number do not appear, the author shows that discharge coefficients of normalized nozzles and Venturis are practically identical when the contraction ratio does not exceed 0.45.
E. Mühlemann, Switzerland

1116. M. Cuenod, The influence of water hammer effects on the governing of hydraulic turbines (in French), Houille blanche 2, 163–182 (Mar.-Apr. 1949).

In this paper the problem of the stability of the automatic control of hydroelectric power units is treated by means of the operational calculus and the Laplace transform. Their basic use in automatic regulation theory is presented first. The classical equations of the revolving masses, the variations of the power of the turbine, the movement of the water in the pressure pipe line, and the boundary conditions at the entrance to and exit from the pipe line, are established and translated into the language of the Laplace transform. Then especially the influence of the variations of pressure upon the stability of the control system is studied, and finally, for an example of a special hydroelectric power plant, the conditions of stability are worked out numerically.

The author claims that these methods permit treating the problem very generally and are valid for high-head, medium-head and low-head power stations. They also allow taking into account such factors which the classical methods had to neglect by reasons of simplicity; for instance, the time lag in the governor system, or phenomena of resonance between a single power station and a strong network to which it is slightly connected.

The method forms in a certain way a bridge between graphic and analytical methods. The bibliography is extensive.
Wilhelm Spannhake, USA

Incompressible Flow; Laminar; Viscous

(See also Revs. 1024, 1113, 1143, 1159, 1197, 1198)

1117. Hans Ertel, A new method for the construction of trajectories in flow fields (in German, with Russian summary), Z. angew. Math. Mech. 28, no. 9, 270–274 (1948).

The paper describes a step-by-step integration procedure for finding the trajectory of a fluid particle in an unsteady field of flow. Both numerical and graphical methods are discussed.
C. C. Lin, USA

1118. H. Schlichting, Lecture series "Boundary layer theory": Part 1—Laminar flows, Nat. adv. Comm. Aero. tech. Memo. no. 1217, 165 pp.; **Part 2—Turbulent flows,** same source, no. 1218, 136 pp. (Apr. 1949); transl. from Zent. wiss. Berichtswesen Luftfahrtforsch. GenLuftZeugmeisters Untersuch. Mitt., W.S. 1941–42.

Part 1 develops the general theory of viscous fluid, and presents many known exact solutions of the Navier-Stokes equations. Stokes' and Oseen's theories of slow flows are given. Next, Prandtl's boundary-layer theory is introduced, and many examples of exact solutions investigated. The method of von Kármán and Pohlhausen is presented in detail. Finally, the paper takes up prevention of separation and the theory of boundary layer with suction. The material is carefully organized and clearly presented.

Part 2 is concerned with (a) the laws of fully-developed turbulent flow, and (b) the origin of turbulence, the first problem being in the foreground. Prandtl's theory of mixing lengths is introduced and applied to pipe flow. Various laws of turbulent flow, concerning velocity distribution and resistance, are studied in detail. Further applications are made to friction drag of a flat plate in longitudinal flow. Gruschwitz's method for the turbulent friction layer in accelerated and retarded flows is presented in detail.

Free turbulence, as in jets and wakes, plane and circular, is considered: Power laws for the increase in width and decrease in velocity with the distance are derived, according to Tollmien, Swain and the author. The determinations of the profile drag from the loss of momentum, due to Betz and Jones, are given.

Finally, the author takes up the problem of the origin of turbulence by regarding it as a stability phenomenon. The method of small oscillations is applied to the stability problem of the laminar boundary layer.

Since the lecture series was given in the winter of 1941, it is unavoidably dated.
S. S. Shu, USA

1119. L. N. Liebermann, The second viscosity of liquids, Phys. Rev. 75, 1415–1422 (May 1, 1949).

The usual second-order equation of hydrodynamics for plane waves in the x direction becomes:

$$\rho(\partial u/\partial t) + \partial p/\partial x = (2n + n')\partial^2 u/\partial x^2$$

where u is the particle velocity, ρ the density, p the pressure, n the usual coefficient of shear vicosity, n' the coefficient of the second or dilatational viscosity which intervenes when a volume of fluid is compressed or dilated without change in shape. As divergent motion is associated with acoustic waves, the dilatational viscosity is an important factor in acoustics.

Following a suggestion of Eckart, who has recently developed a set of second-order equations of acoustics, including the streaming of liquids and gases away from a vibrating source, the author has determined the ratio n'/n from a study of the acoustical streaming for a number of liquids.

The obtained ratios n'/n have positive values and vary widely for all the liquids studied. There is no direct correlation between shear and dilatational viscosity: the latter, for most liquids, is considerably greater than the former.

Values for the dilatational viscosity are also obtained from sound-absorption measurements of liquids; they agree in general with those obtained from the present experiments; therefore the major part if not all of the sound absorption in most liquids is a result of viscous dissipation.

The values of viscosity ratio n'/n show no significant variation with the temperature from 4 C to 25 C. They show a remarkable dependence on frequency, increasing with decreasing frequency; this may be explained by the assumption that the relaxation time is greater for dilatational than for shear viscosity.
Giulio de Marchi, Italy

1120. W. Prichard Jones, Aerodynamic forces on wings in simple harmonic motion, Rep. Memo. aero. Res. Counc. Lond. no. 2026, 28 pp. (Feb. 1945, publ. 1947).

As the pressure distribution (or distribution of bound vortexes) on an oscillating wing of any plan form in linearized theory is proportional to the total time derivative of the jump in the velocity potential on the wing and on the vortex sheet, a relation between the velocity potential and the distribution of bound vortexes is obtained. Representing the discontinuity of the velocity potential by a distribution of doublets on the wing and in the vortex sheet, an integral equation for the doublet distribution is obtained by

putting the normal velocity on the wing equal to the velocity prescribed by the oscillatory motion.

If the distribution of the bound vortexes is expanded in a series of products of simple functions of the spanwise coordinate and functions of the chordwise coordinate, taken from two-dimensional theory (involving Hankel functions) with undetermined coefficients, the first relation delivers an analogous series for the doublet distribution. Substituting into the integral equation, a set of linear equations for the coefficients is obtained. The integrals occurring in this substitution are calculated by a method due to Cigala. After a discussion of some previous theories, the author gives an alternative, which is applied to the calculations of the forces on a flat wing in flexional and torsional oscillation. Comparison with experimental values shows reasonable agreement.

R. Timman, Holland

1121. **O. Holme, Wind tunnel tests of tapered wings with various amounts of dihedral and sweepback,** Roy. Swed. Air Board Rep. Transl. no. 7, 24 pp. (1948).

This report presents the experimental results and analysis of a series of wind-tunnel tests made on tapered wings with various amounts of dihedral and sweepback. The wing model employed incorporated a rectangular center section with tapered outer panels and hinged attachments which permitted adjustment of the two primary variables in the test program. The range of sweepback angles was from −3.3 deg to +9.8 deg. Dihedral angles of 0, 7 and 14 deg were included in the test program. In addition to the usual range of angles of attack from zero lift through the stall, the effect of angles of yaw from −5 deg to +20 deg was studied. All tests were made at an air speed of 45 m/s. In addition to six-component force and moment tests, flow investigations of the stalling characteristics were made by observation of tufts attached to the upper surface of the wings.

The results are presented primarily in the form of the variations of the lift and moment curve slopes with angles of yaw, dihedral and sweepback. Sketches of the flow characteristics indicating stalled areas are also shown for the various wing arrangements and attitudes. The tests appear to be of particular value because of the rather systematic and thorough nature of the investigation, as well as the inclusion of a large range of angles of dihedral.

M. J. Thompson, USA

1122. **W. Perl and H. E. Moses, Velocity distributions on two-dimensional wing-duct inlets by conformal mapping,** Nat. adv. Comm. Aero. Rep. no. 893, 13 pp. (1948).

A method is developed to describe the incompressible potential flow in the leading-edge region of a two-dimensional inlet duct composed of two staggered airfoil sections of arbitrary shape with adjacent concave sides. The actual configuration is approximated by a duct consisting of the two chosen airfoil sections and parallel planes extending from the trailing edges to downstream infinity. The approximate configuration is mapped iteratively on a pair of staggered semi-infinite parallel planes, which in turn are mapped on a half plane and, finally, on a circle. The results are applied, in the first 8% of duct length, to the calculation of velocity distributions on both surfaces of both airfoils for several configurations of various stagger, over-all lift coefficient, and inlet velocity ratio (of the asymptotic value of the concave-side velocity to free-stream velocity).

Suggestions are made for applying the theory to conformal mapping of leading-edge regions of single airfoils and cascades.

The theory yields surface-velocity distributions which are in reasonable agreement with experiments on an unstaggered duct at lift coefficients of 0.3 and 0.6, and inlet velocity ratio 0.5. For the zero-lift condition, the theory for inlet velocity ratio 0.5 compares best with experiment for inlet velocity ratio 0.473.

W. G. Cornell, USA

1123. **S. I. Wiselius, Drag and pressure measurements with plaster spheres in wind tunnels 3 and 4 of the National Aeronautical Research Institute,** Nat. Luchtlab. Amsterdam Rep. no. 950, 1–8 (1947).

The critical Reynolds number, found with 6- and 8-in. spheres, varied inversely with the diameter to the 0.21 and 0.34 powers for the two tunnels. Attention is given to a vibrationless wire suspension, and effects of tightening loads and spindle size are found negligible.

J. M. Robertson, USA

1124. **James V. Robinson, The viscosity of suspensions of spheres,** J. Phys. and Coll. Chem. 53, 1042–1056 (1949).

The viscosities of suspensions of glass spheres with diameters in the range between 10 and 30 microns in two motor oils, castor oil, corn syrup and polyethylene glycol and sucrose solution were measured in a Couette type viscometer. Apparent viscosity data at rates of shear up to 639 reciprocal seconds are given for several volume concentrations up to 65% and the expression $\eta_0 V/(\eta - \eta_0)$ calculated. η_0 is the viscosity of the medium, η that of the suspension and V the volume per cent. This expression is the reciprocal of the constant in the equation $(\eta - \eta_0)/\eta_0 = \eta_{sp} = kV$. The constant k is equal to 2.5 in Einstein's theoretical derivation. The volume V must be corrected for liquid entrapped with agglomerates of particles. The effective volume of the particles S' is defined as total volume per unit volume of solid. The corrected free volume then is equal to $1 - S'V$. The final equation is $V/\eta_{sp} = 1/k - S'V/k$. It is linear in V/η_{sp} and V and the intercept at $V = 0$ is $1/S'$. Plots of V/η_{sp} against V gave straight lines showing that good fit is obtained. The intercept at $V = 0$ is nearly constant for the six liquids tested.

The theory developed is applied to similar published data with equally satisfactory results. The interpretation of S' as the effective volume of the particles is corroborated by the experimentally determined sediment volume: the volume occupied by the suspended particles after complete settling. The data show the two are closely related. No physical significance can be assigned to k.

C. R. Bloomquist, USA

1125. **Frederick H. Imlay, Theoretical motions of hydrofoil systems,** Nat. adv. Comm. Aero. Rep. no. 918, 25 pp. (1948).

Results are presented of an investigation that has been undertaken to develop theoretical methods of treating the motions of hydrofoil systems, and to determine some of the important parameters. Variations of parameters include three distributions of area between the hydrofoils, two rates of change of downwash angle with angle of attack, three depths of immersion, two dihedral angles, two rates of change of lift with immersion, three longitudinal hydrofoil spacings, two radii of gyration in pitching, and various horizontal and vertical locations of the center of gravity. Graphs are presented to show locations of the center of gravity for stable motion, values of the stability roots, and motions following the sudden application of a vertical force or a pitching moment to the hydrofoil system for numerous sets of values of the parameters.

The lateral stability of tandem-hydrofoil systems is briefly discussed, and values of the lateral stability roots are presented for two longitudinal hydrofoil spacings and two vertical locations of the center of gravity.

The analysis indicates that if only the longitudinal motions of a hydrofoil system are of interest, the present theory should provide satisfactory predictions. An adequate theory for the lateral motions, however, must treat the longitudinal and lateral motions

in combination. The conclusions based on the investigation are that a large longitudinal spacing between the hydrofoils, a large rate of change of lift with depth of immersion, and a horizontal location of the center of gravity near the center of the region of stable locations are important contributions in the attainment of desirable characteristics for the longitudinal motion. An appendix gives an outline of the method of theoretical treatment used, and presents methods used in computing the required stability derivatives.

The author has presented a fundamental approach to the stability of hydrofoil systems, and in view of increasing interest in the use of hydrofoil sustention for high-speed planing craft, both displacement type and aircraft, this paper is timely as a basis of departure for many of the problems that will be encountered.

E. G. Stout, USA

1126. T. H. Havelock, Calculations illustrating the effect of boundary layer on wave resistance, Trans. Instn. nav. Archit. Lond. 90, 259–271 (1948).

This paper should be of interest not only to naval architects, but to those concerned with the mechanism of fluid resistance. It provides a plausible, though qualitative, explanation of a discrepancy between experimental determinations of the drag of ship forms and results given by the theory of wave resistance. For the usual form of hull, the theory shows a succession of humps and hollows in the curve of resistance coefficient vs. speed. This feature of the predicted drag curve is evidently the result of coincidences of the boat length with multiples of the wave length. It is stated that such marked oscillations do not appear in the actual drag curves, probably because of an action of viscosity, which is not taken into account in the theory. It is known that viscosity results in the formation of a thick boundary layer or wake in the vicinity of the stern. The author proposes that the effect of such a wake be represented qualitatively by the addition of a small cusp-shaped extension to the actual outline of the boat. The paper shows that if a reasonable allowance of this kind is made and the theory of wave resistance is applied to the modified outline, then the excessive oscillations in the drag curve disappear and a drag variation more closely resembling the experimental curves is obtained.

The analysis naturally opens the way to questions concerning the details of the interaction between the surface waves and the viscous wake. One factor which may require consideration if a more detailed examination of the phenomenon is attempted, is the effect of the boundary-layer turbulence, which greatly magnifies the apparent viscosity of the water and tends to suppress the continuation of waves into the wake region. Although the paper is frankly a simplified treatment, the calculated results leave little doubt as to the general validity of the explanation given.

Robert T. Jones, USA

1127. R. Guilloton, Streamlines on fine hulls, Trans. Instn. nav. Archit. Lond. 90, 48–63 (1948).

The author uses the Michell expression [Phil. Mag. (1898)] for the velocity potential due to the movement of a very fine ship's hull through an inviscid fluid, in connection with an artifice earlier introduced by him, viz., a composition of straight-line generated "wedges" unlimited toward the rear. Completing earlier investigations, he now by these means gives an approximate account not only of the surface-wave profile but also of the streamlines and the lines of constant pressure over the whole surface of the submerged hull. The investigation implies troublesome evaluations of integrals, of which eight tables are given.

Also methods to correct some of the deficiencies of the artificial ship are discussed, and the use of the results as a first step for carrying out corresponding calculations for ordinary hulls by successive approximations is mentioned. An interesting attempt to allow for a flat bottom is to be noted.

Einar Hogner, Sweden

Compressible Flow, Gas Dynamics

(See also Revs. 1138, 1140, 1152)

1128. Walter Tollmien, The direct hodograph method in the theory of the flow of compressible fluids, Reissner Anniv. Vol., 89–110, J. W. Edwards, Ann Arbor (1948).

If a steady potential gas flow is given in the "physical" xy-plane, x and y considered as unknown functions in the hodograph plane satisfy linear partial differential equations. The author calls this method of linearizing the gas-dynamical equations, the "direct hodograph method." (It is not essentially different from the linearization by means of the Legendre transformation.) In this paper only supersonic flows are considered and characteristic coordinates are used in the hodograph plane. Using the direct hodograph method the author discusses (1) limiting lines in general; (2) limiting lines in transonic flow [in this connection Friedrich's result, Rev. 3, **753**, on the nonoccurrence of limiting lines should be mentioned]; (3) the propagation of discontinuities along Mach lines; (4) small disturbances in the (transonic) purely circulatory flow past a circle [cf. Taylor, J. London math. Soc. 5, 224–240 (1930)].

L. Bers, USA

1129. B. Regenscheit, Venturi tube with varying mass flow, Nat. adv. Comm. Aero. tech. Memo. no. 1191, 31 pp. (Mar. 1948); transl. from Deutsche Luftfahrtforsch. Forschungsber. no. 1945.

An ordinary Venturi tube is enclosed in an outside shell in such a manner that the tube-wall axial section looks like a hollow airfoil with a typical airfoil leading edge but a trailing edge which is not closed. Thus, when the Venturi is placed axially in an air stream, the amount of air passing through the center depends, as usual, upon the frictional loss in the tube itself. If this flow through the Venturi is considered from the point of view of the tube-wall section, the flow can be considered a mean flow plus a circulation around the wall. By applying suction through the Venturi support which removes air from the stream through the open trailing edge of the Venturi wall, the circulation can be varied and, hence, the mass flow through the Venturi can be varied. A simple analysis is made by introducing appropriate coefficients, and test results are shown for a number of different geometric variations.

It is found that a considerable variation of flow through the Venturi can be controlled by a relatively small flow through the trailing edge opening. An application to shrouded propellers is made in an appendix. No reference is made to "Fluid dynamics control of fluid flow" [Kenton D. McMann, Proc. Fifth int. Congress appl. Mechanics (1938)] which treats of the control of a large-volume flow by varying a small-disturbance flow.

Howard W. Emmons, USA

1130. Paul Torda, Compressible flow through reed valves for pulse jet engines, Reissner Anniv. Vol., 469–493, J. W. Edwards, Ann Arbor (1948).

In this paper the author presents an analysis of the compressible flow through a pair of reed valves of a pulsejet engine. The flow is treated as one-dimensional, unsteady, and isentropic. Euler's momentum equation, the continuity equation, the isentropic change of state equation, and the dynamic equation of the reed treated as a beam are used to determine the velocity u, the

density ρ, the pressure p, and the cross-sectional area A as functions of the time t and the distance x from the upstream face of the bank of reed valves measured along the centerline of symmetry between the two valves under consideration. By assuming certain unknown functions in x and t to be separable, the author obtains a solution to these equations, containing certain arbitrary constants and time functions which are determined from initial and boundary conditions. These are formulated for both hinged and clamped reed valves in an inverse manner; that is, the velocity $u(0, t)$ is prescribed and the pressure inside the combustion chamber determined from the governing equations. This permits closed-form solutions.

Numerical calculations are given for hinged valves, it being stated that sample calculations for the clamped-valve case and the results for hinged valves imply similar results for both cases. The examples indicate that slender reed valves should be employed if high bending stresses in the reeds are to be avoided, and that hinging the valves does not alter the stresses appreciably.

Paul A. Libby, USA

1131. **I. Irving Pinkel and Harold Shames, Analysis of jet-propulsion-engine combustion-chamber pressure losses,** Nat. adv. Comm. Aero. Rep. no. 880, 11 pp. (1947, published in 1949).

The combustion-chamber pressure losses are analyzed for three types of jet-propulsion engines: cylindrical and annular turbojet engines, and ramjet engines. A combustion chamber of constant cross-sectional area is assumed to be equivalent to these engines in so far as the pressure-loss characteristics of the actual combustion chambers are concerned. The equivalent combustion chamber is divided into two zones, the first representing the fluid frictional resistance, and the second the combustion process.

The total pressure loss is divided into two terms, the loss due to friction, and the momentum loss. These pressure losses are analyzed by using the equivalent combustion chamber, where the friction loss is expressed in terms of the inlet air conditions and the momentum loss is expressed in terms of the air conditions in the combustion zone. The law of the conservation of mass, perfect-gas law and reversible adiabatic relations are used to reduce the analytical expressions to terms of local Mach number and ratio of the specific heats. These functions are then used to plot some rather elaborate pressure-loss charts. The independent parameters which must be selected to utilize the charts are the temperature ratio across the combustion chamber and a quantity $W\sqrt{T_4}/P_4$, where W is the air mass flow, and T_4 and P_4 the temperature and pressure at the inlet of the combustion chamber.

The use of the pressure-loss charts is described in some detail, and the results obtained for the computed pressure losses in the three types of engines are compared with experimental results; the computed pressure-loss ratios agreeing to within 7% of the experimental values. From this type of analysis it is possible to compare the pressure-loss characteristics of various designs of combustion chambers and to select certain parameters for optimizing the performance of a particular engine.

Freeman K. Hill, USA

1132. **J. M. Burgers, On the influence of gravity upon the expansion of a gas. I,** Nederl. Akad. Wetensch. Proc. 51, 145–154 (1948).

The problem is that of the expansion of a semi-infinite vertical column of perfect gas into a vacuum with the upper partition suddenly removed at the instant $t = 0$. The gas column is kept in isentropic equilibrium against a constant gravitational force for $t < 0$. The ratio of specific heats is taken as 5/3. The problem is solved by Riemann's characteristics method. It is shown that the flow develops shocks, and then the solution breaks down because of the change in entropy. Aside from this uncertainty, the solution obtained shows that the gas settles down to a new equilibrium state, and that the main body of gas moves upward only slightly between the initial and the final equilibrium states.

H. S. Tsien, USA

1133. **Fritz Kai, A contribution to the theory of waves in free gas jets** (in German), Z. angew. Math. Mech. 28, 80–85 (Mar. 1948).

The author states, on the basis of two numerical examples, that wave patterns of free jets at supersonic speed are not necessarily periodic, even in flows with expansion. The periodic jet pattern is destroyed by shock waves arising from the envelope of Mach lines, which, in turn, arise by reflection of the expansion waves impinging on the jet boundary. From the presence of oblique shock waves, there follows that supersonic jets tend to dissipate also in nonviscous gases.

Carlo Ferrari, Italy

1134. **I. Irving Pinkel, Equations for the design of two-dimensional supersonic nozzles,** Nat. adv. Comm. Aero. Rep. no. 907, 24 pp. (1948).

Analytical expressions based on the method of characteristics are given for the determination of the wall coordinates of a certain type of two-dimensional supersonic nozzle. The nozzle has a straight sonic line followed by an expansion to a uniform flow at a given supersonic Mach number. This is converted to a source flow joined by a straightening portion, which gives a uniform flow at the required working-section Mach number. It is shown that the source flow is not possible if the maximum slope of the divergent portion of the nozzle exceeds an angle of 31 deg to the nozzle axis. Numerous methods are described for the conversion of the uniform sonic flow to uniform flow at a slightly higher Mach number.

Numerous functions are tabulated to assist in the computations, but some of the values given (e.g., Mach angle) are incorrect. (Compare with *Gas Tables* by Keenan and Kaye, Wiley, 1948.)

The problem of the design of the subsonic portion of the nozzle to give a straight sonic line is not discussed.

G. M. Lilley, England

1135. **Clinton E. Brown and MacC. Adams, Damping in pitch and roll of triangular wings at supersonic speeds,** Nat. adv. Comm. Aero. Rep. no. 892, 9 pp. (1948).

The aerodynamic damping coefficients of triangular wings in pitching and rolling oscillations at supersonic speeds are computed by the linearized quasi-steady techniques. For the narrow triangular wing (subsonic leading edges) the perturbation potential is represented by a doublet distribution over the wing, and the doublet strength is found by solving the integral equation defined by the boundary conditions. For the wide triangular wing, source superposition methods are used. Pressures are computed from the perturbation potential φ by the steady-state linearized formula, $\Delta p = -\rho U \varphi_x$ where the main stream of velocity U is parallel to the x-axis. The boundary conditions used require that the normal component of the perturbation velocity vary linearly in the spanwise direction (for the rolling motion) or in the chordwise direction (for the pitching motion).

The computation is properly carried out and the results are correct within the limitations of the basic theory. This quasi-steady procedure has been widely used; however, it is the opinion of the reviewer that this basic procedure is of dubious accuracy. In order to obtain results consistent with the linearized nonsteady supersonic theory, it is necessary to retain at least the first-order time-derivative terms in the perturbation potential differential equation and in the pressure equation, $\Delta p = -\rho(\varphi_t + U\varphi_x)$

In addition, some terms in the boundary condition for the pitching motion are omitted in the simplified theory. Application of the complete linearized theory to the two-dimensional wing by Garrick and Rubinow [same source, tech. Note 1158] has shown that the quasi-steady theory may even predict the wrong sign for the damping coefficient in pitch. A more nearly complete discussion of the pitching motion of delta wings has been given by Miles [J. aero. Sci. 16, p. 574 (1949)]. As an example of the importance of the neglected terms, the complete linearized theory shows that the result given by Brown and Adams for the coefficient of damping in pitch for pitching about the $^2/_3$-chord point should be multiplied by $(2M^2 - 3)/(2M^2 - 1)$, where M is the flight Mach number. H. J. Stewart, USA

1136. J. Nicolas and H. Audic, Computation tables and diagrams for oblique shock waves in two-dimensional flow (in French), Off. nat. Étud. Rech. aéro. Rep. no. 11, pp. 1–13, 5 charts (1948).

1137. W. Kai, Application of the computational treatment of shock waves and vortex flow to a body composed of a cone and a cylinder, Hdqtrs. Air Mat. Comm. Dayton Transl. no. A9-T-16, 32 pp. (Jan. 1949); transl. from Archiv 44/14, Peenemuende.

The Taylor-Maccoll flow, the expansion wave, and stepwise supersonic-flow computation, using a Mach-line lattice, are applied to the title problem, without neglecting vorticity behind the curved portion of the shock. A vorticity term is included in the characteristic hodograph equations. Two numerical examples are worked out for M = 1.8 and 3.24. Ed.

Turbulence, Boundary Layer, etc.

(**See also Revs. 1118, 1126, 1129, 1198, 1199**)

1138. W. Mangler, Relation between the plane boundary layer and the one with rotational symmetry in compressible fluids (in German), Z. angew. Math. Mech. 28, no. 4, 97–103 (1948).

The author gives a general transformation rule which relates a boundary-layer problem for an axially symmetric case to a corresponding two-dimensional problem. Let s be the length of arc along the body and let n denote length of the normal on s from a point; $r = r_0(s)$ is the equation giving the body contour. Define $\bar{s} = {}_0\int^s (r_0^2(s)/L^2)ds$, $\bar{n} = r_0(s)n/L$, $\bar{\psi}(\bar{s},\bar{n}) = L^{-1}\psi(s,n)$ (stream function), $\bar{p}(\bar{s}) = p(s)$ (pressure at the surface), $\bar{i}(\bar{s},\bar{n}) = i(s,n)$ (enthalpy), $\bar{\rho}(\bar{s},\bar{n}) = \rho(s,n)$ (density) and $\bar{\mu}(\bar{s},\bar{n}) = \mu(s,n)$ (viscosity). The boundary-layer equations for the axially symmetric flow are then shown to be identical with the equations for two-dimensional flow.

Hence the solution of an axially symmetric case for a given pressure distribution $p(s)$ and contour $r_0(s)$ is reduced to the solution of a two-dimensional problem with $\bar{p}(\bar{s}) = p(s)$ and $\bar{s} = {}_0\int^s (r_0^2(s)/L^2)ds$. The important relation between the wall shearing stress in both cases comes out to be

$$\bar{\tau}_0^* = \tau_0^*[{}_0\int^s r_0^2(s)ds/sr_0^2(s)]^{1/2},$$

where $\tau_0^* = (\tau_0/\rho_a V^2)(sV/\nu_a)^{1/2}$, and where ρ_a and ν_a denote the values of density and kinematic viscosity in the free stream of velocity V. Several applications are given.

H. W. Liepmann, USA

1139. R. T. Shiells, Small model experiments and viscosity effects, Trans. Instn. nav. Archit. Lond. 90, 334–345 (1948).

The author deals with the scale effect governing the influence of viscosity on the residuary resistance and on the wave formation. He repeats, on a 4-ft scale, some tests made earlier by Wigley with a 16-ft unsymmetric model [same source, 2130 B (1944)], viz., (a) measurement of the resistances with fine and full ends leading, and (b) plotting of the wave profile by photography. The small model (with two roughened strips of paint and sand each 2% of the wetted surface applied one at each end), running through a range of Reynolds numbers from 6×10^5 to about 1.3×10^6, operates within the transition range of plank friction.

The main results for the residuary resistance formed by means of the Schoenherr frictional law are the following: Deviation from the Froude similarity law is found as the humps and hollows of the 4-ft model are less pronounced than those of the larger model, due to the comparatively greater viscosity effects in the smaller scale. The similarity is better developed with the fine end leading, than in the opposite direction of run. The residuary resistances in the two opposite directions become equal in both scales above approximately the same lower limit of Froude's number, as earlier stated by Wigley for the 16-ft model. The author concludes that turbulent flow is produced also by the small model, and that the viscosity effect on the wave formation is nearly independent of scale. A fuller analysis of the influence of depth would have been welcome to the reader. The resistance runs have taken place in a swimming bath, the model being towed by an apparatus operated by gravity. Einar Hogner, Sweden

Aerodynamics of Flight; Wind Forces

(**See also Revs. 1120, 1121, 1135, 1145**)

1140. Alex Goodman and Lewis R. Fisher, Investigation at low speeds of the effect of aspect ratio and sweep on rolling stability derivatives of untapered wings, Nat. adv. Comm. Aero. tech. Note no. 1835, 37 pp. (Mar. 1949).

Untapered wings of different aspect ratios (1.34; 2.61; 5.16) and different sweep angle (0; 45; 60 deg) were tested in the rolling equipment of the Langley stability tunnel. The Reynolds number based on chord was 1.4 to 2×10^6, the Mach number 0.13 to 0.17. The measured rolling-stability derivatives are compared with those resulting from simple theories, and empirical corrections of the existing formulas are suggested.

I. Flügge-Lotz, USA

1141. Alfred Gessow and Kenneth B. Amer, An introduction to the physical aspects of helicopter stability, Nat. adv. Comm. Aero. Tech. Note no. 1982, 35 pp. (1949).

The authors do not attempt to add to the knowledge of helicopter stability but seek to provide engineers interested in rotating-wing aircraft, but with no specialized training in stability theory, some understanding of the factors that enter into the flying qualities of the helicopter. They give stability definitions and discuss in sound, though easily understandable form, the elements of (a) rotor control; (b) damping in pitch and a simple formula for damping; (c) flight stability, that is, stability due to changes in translational velocity; (d) variation of rotor moment and force as related to changes in the fuselage angle of attack; (e) static stability, that is, stability referring to angular displacement; (f) dynamic stability, the study of oscillations when disturbed, a topic that is dealt with by analogy with the dynamic stability of the airplane; (g) helicopter motion following a disturbance including a formula for the period of oscillation of a hovering helicopter; (h) static stability which like dynamic stability is related to the static stability of the airplane; (i) the effect of stability parameters on divergence of oscillation.

The paper summarizes the present status of helicopter stability:

(1) In hovering, the helicopter possesses neutral static stabil-

ity with respect to attitude changes but has positive static stability with respect to changes in translational velocity.

(2) When disturbed from a hovering condition, the period or the resulting motion depends upon moments due to changes in speed (flight stability) and moments due to angular velocity (damping in pitch or roll).

(3) In forward flight, static stability with attitude change as well as static stability with speed change must be considered.

(4) In forward flight, the helicopter rotor is statically stable with speed and statically unstable with angle of attack.

(5) The static stability of the helicopter in forward flight is unaffected by a center-of-gravity shift if no moments are contributed by components other than the rotor.

(6) Dynamic instability in forward flight can be reduced by the addition of positive static stability with angle of attack, by increasing the damping in pitch, or by a sacrifice in flight stability.

Alexander Klemin, USA

1142. **Royal Aeronautical Society Data Sheets: Performance,** Royal Aeronautical Society, London, July 1949, 24 leaves (16 graphs). Loose-leaf binder, 10.5 × 13.5 in., price unstated.

This is the first series of RAS data sheets intended to provide the designer with information on performance reduction. The second series will cover performance estimation. The sheets consist of three sections: general, turbojets and turboprops.

The general section is concerned with corrections to instrument readings. For turbojets and turboprops graphical methods are given for the reduction of level-speed, rate of climb, range and endurance. For turbojets, additionally, an analytical method for the rate-of-climb reduction is presented.

The general arrangement of the matter within a sheet was described in Rev. 2, **608**. Ed.

1143. **Paul Dupleich, Rotation in free fall of rectangular wings of elongated shape,** Nat. adv. Comm. Aero. tech. Memo. no. 1201, 99 pp. (Apr. 1949); transl. from Publ. sci. tech. Secrét. État Avia. no. 176 (1941).

This paper describes the results of a careful experimental study of the problem of the rotation in free fall of thin, flat-plate wings. Attention is concentrated on wings of rectangular plan form, although several other shapes are also investigated briefly. Careful observations, by means of a photographic technique, enabled the tracing of the trajectories and rotary behavior of a series of wings in free fall in atmospheric air; a limited number of tests were also conducted in water. The data are reduced to certain empirical equations. A brief analytic study is also included to illuminate somewhat the mechanism of the motion.

Martin Goland, USA

1144. **E. I. Auterson, Wind tunnel tests on clusters of parachutes,** Rep. Memo. aero. Res. Counc. Lond. no. 2322, 8 pp. (1949).

Tests are described on nonvented parachutes of 2-ft nominal inflated diam and 2-ft or 4-ft rigging lines, designed to have individual critical speeds at 130 ft per sec. The tests indicate that "crowding" of the chutes decreases drag and opening speeds, and increases stability of the cluster. The use of strops to prevent crowding results in increased drag to almost that of the sum of the individual drags. A similar effect of strops on the opening speed was noted. With strops the stability was decreased and a tendency to intertwine was noted. This latter effect was reduced by increased porosity.

H. P. Liepman, USA

Aeroelasticity (Flutter, Divergence, etc.)

1145. **H. M. Lyon, A method of estimating the effect of aeroelastic distortion of a swept-back wing on stability and control derivatives,** Rep. Memo. aero. Res. Counc. Lond. no. 2331, 10 pp. (1949).

A method of estimating the aeroelastic effects of a swept wing on the stability derivatives is discussed. The method consists of the determination of the mode of distortion of the wing on the basis of beam theory for aerodynamic loads on the undistorted wing. The aerodynamic loads for the distorted wing are then determined, and the magnitude of the deflections corresponding to these loads are obtained by the principle of virtual displacement on the assumption that the mode does not change. A second approximation to the mode can then be made and the process can be continued if desired.

The method is proposed as a basis for discussion and further investigation, and no numerical evaluations are presented so that an estimate of the effectiveness of the method cannot be made.

M. V. Barton, USA

1146. **A. I. van de Vooren, A method to determine the change in flutter speed due to small changes in the mechanical system,** Nat. Luchtlab. Amsterdam Rep. V. 1366, 71–74 (1947).

In this paper a method is developed to evaluate the influence on flutter speed of small structural changes of the airplane, without performing another complete flutter calculation. Use is made of the results obtained from the flutter calculation of the airplane in its original state.

From author's summary by N. O. Myklestad, USA

1147. **A. I. van de Vooren, The change in flutter speed due to small variations in some aileron parameters,** Nat. Luchtlab. Amsterdam. Rep. V. 1380, 53–58 (1947).

This paper contains diagrams showing the change in flutter speed due to small variations in aileron static balance, in aileron moment of inertia, in the ratio of aileron chord to total chord and in the position of the aileron hinge axis. The principal conclusions were: Underbalancing of the aileron has apparently a very unfavorable influence on the flutter speed. In general, an increase of the aileron moment of inertia is unfavorable, but variation of the ratio of aileron chord to total chord has only a slight influence. Shifting of the aileron hinge-axis forward leads to a lower flutter speed in nearly all cases. N. O. Myklestad, USA

1148. **R. Weber, Orthogonalization of vibration modes of an airplane, measured in ground resonance tests** (in French), Off. Nat. Étud. Rech. aéro. Rep. no. 29, 23 pp. (1949).

The orthogonality of measured modes is considered to be spoiled by inaccuracies and by structural damping (said to be omitted in flutter calculations). A correction procedure is proposed. J. H. Greidanus, Holland

1149. **Heinz Bernhard, On bending vibrations of turbine blades with a T-shaped base** (in German), Technik 3, 152–154 (Apr. 1948).

It is pointed out that the assumption of rigid fastening at the base of a turbine blade of this particular design leads to great error in the calculation of the natural frequencies of vibration. In a practical example it is shown that the fundamental frequency is only one-tenth of that found by assuming rigid clamping at the base. The author says that this is verified by experiment. His boundary conditions are of the form $Y''(1) = Y'''(1) = 0$ at the free end; $Y''(0) = (1/k)\, Y'(0)$, and $Y(0) = bY'''(0)$ at the

base, the constants b and k being derived from the elastic properties of the blade. The characteristic equation is solved by an approximate method. However, it has been assumed that the flexural rigidity EI of the blade is constant. The author does not indicate to what extent this affects the accuracy of the solution.

Edward Saibel, USA

Propellers, Fans, Turbines, Pumps, etc.

(See also Revs. 1116, 1130, 1131, 1141, 1163)

1150. **John T. Sinnette, Jr., Analysis of effect of basic design variables on subsonic axial-flow-compressor performance,** Nat. adv. Comm. Aero. tech. Memo. no. 901, 21 pp. (1948).

A blade-element theory for axial-flow compressors has been developed and applied to the analysis of the effects of basic design variables such as Mach number, blade loading, and velocity distribution on compressor performance. A graphical method that is useful for approximate design calculations is presented. The relations among several efficiencies useful in compressor design are derived and discussed. The possible gains in useful operating range obtainable by the use of adjustable stator blades are discussed, and a rapid approximate method of calculating blade-angle resettings is shown by an example.

The relative Mach number is shown to be a dominant factor in determining the pressure ratio. Considerable increase in pressure ratio over that for conventional designs can be obtained by producing a velocity distribution that gives relative inlet Mach numbers close to the limiting Mach number on all blade elements. With a given inlet Mach number, the pressure ratio obtainable across a blade row increases, and the specific mass flow decreases as the ratio of mean whirl velocity to axial velocity increases for the high-efficiency range of this velocity ratio (that is, near 1). For compressor designs with a definite Mach number limitation, the velocity distribution in the inlet stage is important.

R. C. Binder, USA

1151. **R. Waeselynck, Some considerations on gas turbines** (in French), Bull. Ass. tech. Marit. Aéro. 46, 711–747 (1947).

The first part of this paper discusses a gas-turbine cycle consisting of two isotherms and two isobars, and the requirements for approaching such a cycle in reality by using one or two intercoolers between compressors and two combustion chambers resulting in at least one reheating stage. The author discusses the effects of the strength of material as function of temperature and temperature difference, pressure losses, external heat losses and mechanical losses. For the above assumptions the author presents the necessary formulas for calculating the total efficiency, and comes to the conclusion that the gas turbine is equal to or better than the Diesel engine, provided the turbine exit temperature is on the order of 600 C.

In the second part of the paper the author discusses a special application of the previous cycle to the propulsion of a naval vessel requiring high efficiency at cruising speed and also at maximum power. For cruising, the open-cycle gas turbine has a high-pressure turbine driving two compressors with intercooling and two combustion chambers for reheat of the gas to the propulsion turbine which drives only the shaft. The exhaust from the turbine goes to the regenerator. For maximum power an additional low-pressure compressor and turbine are added to the cycle so that the cruising compressor and turbine plant now operate at elevated pressure levels. During maximum power the speed of the cruising turbine compressor set is also increased about 15%. The total pressure ratio for cruising is about 4 and at maximum power about 20. With a regenerator efficiency of 90% the total efficiency for cruising is 32% and for maximum power 30.8%.

The paper is followed by a discussion which critically reviews the high regenerator effectiveness, some of the other assumptions, and compares the results with a closed-cycle gas turbine and combination vapor-gas cycles. The paper presents an interesting solution for applications where high efficiency is required and the part load is about $^1/_8$ of the full load, i.e., the cruising speed is about half of the maximum speed of the vessel.

H. E. Sheets, USA

Flow and Flight Test Techniques

(See also Revs. 1107, 1119, 1123, 1134, 1139, 1196, 1199, 1200)

1152. **K. Kessels, Flow measurements** (in German), Arch. Eisenhüttenw. 20, no. 3/4, 79–106 (Mar./Apr. 1949).

This paper constitutes a comprehensive analysis of the method of flow measurement by means of standard orifices and nozzle-type Venturis. It is based mostly on data found in the latest edition of the German standards of flow measurement (VDI Durchfluszmessregeln DIN 1952, 1948) and on information available to the Düsseldorf Heating Plant.

Two types of pressure difference devices only are considered: the standard orifice, which is said to be used in the majority of cases, and the nozzle-type Venturi, which is resorted to when the tolerable pressure loss is small. Most of the information presented is not new, but it is made available in a readily usable form. Distinct formulas are given for dry or moist gases, air, water vapor, or water.

The most interesting feature of the paper is probably a thorough discussion of the tolerances and errors to be expected in a variety of cases where standard conditions of construction, installation and operation cannot be adhered to. The article also contains various illustrative examples and detailed specifications on the installation and maintenance of the devices.

It is to be regretted that no effort has been made to compare the German code of practice with other recent standards.

André L. Jorissen, USA

1153. **Richard S. Cesaro and Norman Matz, Pressure-sensitive system for gas-temperature control,** Nat. adv. Comm. Aero. Rep. no. 896, 7 pp. (1948).

As stated in the authors' summary, a thermodynamic relation is derived and simplified for use as a temperature-limiting control equation involving measurement of gas temperature before combustion, and gas pressures before and after combustion. For critical flow in the turbine nozzles of gas-turbine engines, the control equation is further simplified to require only measurements upstream of the burner. Hypothetical control systems are discussed to illustrate application of the control equations.

This reviewer believes the pressure-sensitive systems described in this report for gas-temperature control of turbojet engines to be quite valid on the basis of one-dimensional flow theory. An objection to the development given here is that it is known that flow unmixedness in the combustion exhaust gases can cause considerable error in utilizing measurements from pressure pickups. However, the pertinent variables to be measured are accounted for in a correct fashion, and for most practical purposes the approximations used in deriving the control equations are quite accurate if applied to a specific engine operating under a reasonable range of outlet temperature.

The authors are to be commended on the theoretical development of an interesting hydraulic control mechanism. Inherent lags due to flows in the system may cause oscillatory responses.

but this reviewer believes such oscillations may be overcome by the use of suitable pressure dampeners in an actual instrument. An actual working instrument for control purposes based on theory presented here appears entirely possible.

Emerson L. Kumm, USA

1154. **D. A. Efros, Construction of streamline flows by the method of electric modeling** (in Russian), Nauk SSSR Ser. teckh. Nauk 9, 1061–1068 (Sept. 1947).

The node voltages u of a square-lattice, pure-resistance passive network (A_x^{-1}, A_y^{-1} are the resistances per unit length) satisfy the equation $(A_x u_x)_x + (A_y u_y)_y = 0$, if no currents are applied. If $A_x = A_y =$ const, u is an incompressible-flow potential. The electric boundary conditions for this familiar analogy are stated for symmetric and asymmetric cavitating flows around a simple open arc. Free boundaries are handled by means of successive trial-and-error reductions.

Ed.

Thermodynamics

(See also Rev. 1151)

1155. **Karl K. Darrow, Atomic energy,** New York, John Wiley and Sons, Inc.; London, Chapman and Hall, Ltd.; 1948, vi + 82 pp., 12 figs. Cloth, 5.5 × 8.5 in., $2.

This is a series of four Norman Wait Harris lectures delivered at Northwestern University in 1947. It is "a narrative of the basic facts most pertinent to the transformation of energy of rest mass into energy of motion and energy of heat . . . There is a good deal about radioactivity, but only as much as seems essential to an understanding of . . . the pile."

Ed.

1156. **A. Pommellet, On thermodynamic principles** (in French), Bull. Assn. tech. Marit. Aéro. 46, 749–771 (1947).

Some basic concepts of thermodynamics are critically analyzed (heat, reversible process as a succession of stages of equilibrium, entropy). The paper is not exhaustive. The discussion, due to some of the best French thermodynamicists, appears very interesting.

Gino Moretti, Argentina

1157. **Werner Matz, The thermodynamics of heat and mass exchange in unit operations (Die Thermodynamik des Wärme- und Stoffaustausches in der Verfahrenstechnik)** (in German), Frankfurt/Main: Verlag Dr. Dietrich Steinkopf, 1949, xvi + 355 pp., 114 figs. Paper, 6 × 9 in., approx. $6.50.

This book on the thermodynamics of binary and ternary systems is a valuable contribution to the literature on applied thermodynamics. The author's original ideas make possible a uniform presentation of such diversified processes as evaporation, distillation, absorption, and extraction. Graphical methods are extensively employed, and the literature up to 1940 has been utilized in the text.

In the initial chapter of 78 pages the application of the elementary thermodynamic concepts to the equilibrium of binary systems and the principles of dimensional analysis are discussed. Thereafter, from a theorem of projective geometry, the "harmonic polar triangle" is introduced and is shown to be a particularly useful tool for the graphical analysis of heat and mass exchange between two phases in counterflow. The use of this method and of similarity principles is one of the merits of the book.

The chapter on heat exchangers includes the heat transfer in the unsteady state and a synopsis of the theories of similarity between friction and the flow of mass and heat. The remaining five chapters deal individually with the processes of evaporation, distillation, adsorption, absorption, and extraction. In each case, theory as well as industrial application are discussed and numerical examples are given. Analytical and graphical methods are employed, according to best advantage.

In an appendix, the author applies the methods of vector analysis to the two laws of thermodynamics, and shows that irreversibilities in a thermodynamic process are analogous to the dissipation of energy by vortexes in a mechanical process. With this result, the author believes to have found a clue to the meaning of irreversibility; however, it is difficult to see the physical reality behind this analogy. In the present stage, this discussion may be regarded rather as an attempt than as a final answer.

E. F. Lype, USA

1158. **Jacques Duclaux: Compressibility of gases and association,** C. R. Acad. Paris 226, 72–74; **Compressibility and condensation of gases,** same source 226, 2034–2036; **Compressibility of gases as a function of the temperature,** same source, 227, 1124–1126 (1948)

The author has previously shown [J. Phys. 8, pp. 94 and 277 (1947)] how to derive an equation of state from the assumption that gas molecules associate. In the first paper this assumption is refined by allowing combinations of 2^n molecules ($n = 1, 2, 3, \ldots$). The pv values thus calculated (no general formula is given) are in relative error of the order of 10^{-3}, and more accurate than those of van der Waals.

The second paper hints that the equation of state may be of the type $P(V - b)/RT = F[a \log(V - b)]$, and applies the concept of association to the theory of condensation.

The third paper infers from the cited form of the equation of state that any two isothermals can be made congruent by changing the unit on the volume axis for one of them. Measured isothermals agree with this prediction. It follows that the condensation law does not depend on the temperature, and contains one constant dependent on the nature of the gas.

Ed.

Heat Transfer; Diffusion

1159. **George V. Parmelee and Richard G. Huebscher, Forced convection heat transfer from flat surfaces, Part I—Smooth surfaces,** Trans. Amer. Soc. heat. vent. Engrs. 53, 245–284 (1947).

Values of unit thermal conductance measured for a range of Reynolds modulus from 19,000 to 1,200,000 are given for a flat plate in air flow. Comparisons with other measurements are given. The effects of unheated starting sections are considered.

Myron Tribus, USA

1160. **H. Glaser, The efficiency of heat exchangers** (in German), Chem. Ing.-Techn. 21, no. 5/6, 95–99 (Mar. 1949).

Usually the "efficiency" of a heat exchanger is defined by the ratio of the effective heat transfer to the maximum possible heat transfer. This ratio is not a sufficient criterion for the selection of an optimum exchanger which has to work under specified conditions in a given cycle. Therefore the author proposes to characterize the exchanger by means of the decrease in cycle performance due to the total of exchanger losses. A number of examples show how to calculate the efficiency according to the above basic concept.

Despite the advantages of the new definition this reviewer feels, that in many cases where exchangers are used in complicated cycles the concept is not practical, and it seems doubtful whether it can be put into general use.

E. Haenni, Switzerland

1161. Ernst Schmidt, The design of contra-flow heat exchangers, Proc. Inst. mech. Eng. 159, no. 45, 351–362 (1948).

The problem of contraflow heat exchangers for gases in turbulent flow is treated, in a general way, using the analogy between heat transfer and friction. This analogy suggests a new nondimensional heat-transfer coefficient **St** = (heat-transfer coefficient)/[(velocity)·(specific weight)·(specific heat)], the reciprocal of the **Pr** number. The main dimensions of the apparatus, and their extreme values, are given in terms of the design heat exchange, the total temperature drop and energy loss, and the properties of the fluid.

The paper is followed by a penetrating discussion of the following topics: the physical significance of the numbers **Re**, **Pr**, **Nu** and **St**; the difference between the mechanical and the aeronautical engineer's requirements with regard to length of life, weight and space; heat exchangers for gas turbines (aeronautical, contraflow, crossflow).

D. Jacovleff, Belgium

1162. H. B. Nottage, Thermal properties of building materials used in heat flow calculations, Trans. Amer. Soc. heat. vent. Engrs. 53, 215–243 (1947).

The author has collected data from about 30 references on thermal conductivity k, density ρ and unit heat capacity c_p for building materials grouped as follows (g—data in good order; f—data in fair order; ei—data could be extended and improved): wood (27g), building and insulating boards and insulating materials (27f), masonry materials (16ei), concrete, plaster and mortar (14ei), glass (6g), roofing materials (16ei). The numbers in parentheses indicate the numbers of species of the groups for which k, ρ, c_p, $a = k/\rho c_p$ and $b = k\rho c_p$ (the latter is used in the Mackey and Wright design charts, same source, Res. Rep. no. 1255) are tabulated. All are expressed in British units.

Comments on the data are given. The data are compared with the empirical equations of Mackey and Wright for use in rough estimates on nonmetallic dry solids, classified by density and origin (vegetable, mineral or either). Graphs of k, b and a show that nearly all points are within 50% from these curves, so that they can serve as convenient aids in estimating thermophysical properties of materials for which data are lacking.

For wood the author gives formulas for c_p, S (specific gravity) and k as a function of S_d (S when over-dry), S_g (S when green), M (moisture content) and t (temperature) and graphs of $M = M$ (t, % relative humidity); $\rho = \rho$ (M, woods); $c_p = c_p$ (M, t); $k = k(M, S_g)$; $b_{70} = b_{70}(M$, woods at 70 F); $a_{70} = a_{70}(M$, woods at 70 F); $f = f(M, t)$; f is a temperature-correction factor: $f = b/b_{70} = a_{70}/a$. A table of S_g and S_d for 25 species of trees is given.

H. Wijker, Holland

1163. J. J. Samolewicz and G. A. Macaulay, Notes on some charge heating anti-icing tests with an axial flow turbo-jet, Nat. Res. Counc. Canada, Rep. no. ME-173, 31 pp. (Dec. 1948).

The results are given of five test runs in which ice formation on a Jumo 004 engine was prevented by adding heated air to the inlet. Observations of ice formation and stoppage of the engine are reported. The tests were run on a thrust stand during the winter, using water sprays for ice formation.

Myron Tribus, USA

Acoustics

(See also Revs. 1046, 1119)

1164. Leo L. Beranek, Acoustic measurements, New York: John Wiley & Sons, Inc.; London: Chapman & Hall, Ltd., 1949, vii+911 pp. Cloth, 5.5 × 8.5 in., $7.

Classical acoustics was a branch of mechanics whose theoretical foundation was well established by the end of the nineteenth century. Rayleigh and his contemporaries were handicapped, however, by lack of techniques of measurements and little knowledge of the functioning of the ear. The chief contribution of the twentieth century has been to reduce tremendously both of these limitations by means of electrical measurements. At the same time, acoustics has spread over into such fields as communications, architecture, and psychoacoustics.

It is rather remarkable that, in spite of the tremendous development of electroacoustic apparatus, no book has been published up to the present which adequately describes these new techniques of measurement. The author has now undertaken this ambitious task and accomplished it outstandingly well. His professional experience makes him unusually qualified for this undertaking. A brilliant graduate of the Cruft Laboratory at Harvard University, he organized the Electroacoustic Laboratory at Harvard which, with the Psychoacoustic Laboratory in the same institution, helped solve many of the communications and other acoustic problems of World War II. At the same time these groups helped extend considerably the frontiers of fundamental knowledge in acoustics. At present, the author is technical director of the Acoustics Laboratory at the Massachusetts Institute of Technology.

Not all the subject matter of this 911-page book can be reviewed here. The author states in his preface, "This book was intended primarily as a reference for graduate students and workers in the field of acoustics .... : the acoustic physicist making fundamental laboratory measurements, the communications engineer measuring and evaluating the performance of acoustic communications systems, the psychologist performing measurements involving the human hearing mechanism, the otologist studying hearing defects, and, finally, the industrialist applying acoustic measuring techniques in manufacturing processes."

It is believed by the reviewer, however, that application of some of the techniques described will also have peripheral effects on many branches of dynamical mechanics such as shock and vibration and fluid mechanics. Many of the measurement techniques being developed in these fields parallel the earlier developments in acoustic measurements. Of particular interest to workers in mechanics are sections which treat such subjects as the following: (A) acoustical terminology, (B) properties of the propagating medium; (C) calibration of transducers; (D) piezoelectric microphones; (E) microphones in winds; (F) measurement of frequency; (G) definitions and measurements of mechanical and acoustical impedance; (H) high-intensity sound sources; (I) response of various types of electrical measuring devices to nonsinusoidal excitation; (J) frequency analysis of steady-state sounds; (K) calibration of analyzers; (L) transient response of transducers; (M) acoustical absorption of materials; (N) relation of absorption to fundamental properties such as porosity and flow resistance; and (O) the measurement of sound transmission through building structures.

The author is to be highly complimented on his very exhaustive book which will undoubtedly be a handy reference to many physicists and engineers.

Howard C. Hardy, USA

1165. H. Knotzel and L. Knotzel, Absorption and dispersion of sound in oxygen (in German), Ann. Phys. Leipzig 2, no. 7/8, 393–403 (June 1948).

In this article the authors present the results of their measurements of absorption and velocity dispersion in pure O_2, in O_2 with admixtures of water vapor, and in O_2 with small amounts of NH_3, all at 19 C and at atmospheric pressure. The authors used a resonance method. Suitable concentrations of these added gases

in the oxygen produce shifts of the dispersion region into the audible range (0.6 to 4.5 kc). For pure O_2 the absorption measurements yield a relaxation time for thermal equilibrium which is certainly not less than 3 milliseconds. A functional relationship is set up to express the manner in which the frequency for maximum absorption depends on the added moisture or on the added NH_3. The dispersion measurements which were made with high precision ($\pm 3 \times 10^{-5}$) yield the values 1.3955 for c_p/c_v at zero pressure and very low frequency, and 1.4000 for very high frequency. These values are in complete agreement with theory. Seven figures and three tables are included. The results differ somewhat from earlier results by Kneser and Knudsen and by Schmidtmüller. W. H. Pielemeier, USA

1166. C. E. Mulders, Ultrasonic absorption in water in the region of 1 mc./s., Nature Lond. 164, no. 4165, 347–348 (1949).

The paper is devoted to a method for measuring attenuation in water and other liquids at such low frequencies that the standard ultrasonic pulsing methods are not applicable. A reverberation method is used. The liquid is placed in an aluminum tank and sound waves in the frequency range from 750 kc to 1500 kc are set up in the tank by means of a quartz crystal attached to the tank by a drop of oil. The generating sound is then turned off and the rate at which the sound energy dies down is determined by a second crystal of rochelle salt attached to an amplifier and recorder.

The sources of dissipation are the absorption in the liquid, loss caused by friction along the walls and bottom of the tank, acoustic radiation from the liquid surface, and losses caused by conduction to the crystal surfaces attached to the bottom of the tank. Only the second term turns out to be important. By varying the dimensions of the tank this term can be evaluated and the true attenuation determined. For water the results from 750 kc to 1500 kc agree well with those obtained at higher frequencies by a pulsing method. W. R. Mason, USA

1167. Per V. Bruel, Measurements of sound insulation in buildings (in Swedish), Trans. Chalmers Univ. Technol. Sweden no. 86, 191 pp. (1949).

This work gives a considerable number of results concerning measurements of sound insulation in buildings. Graphs are shown of the insulation vs. air-borne sound as function of frequency of 49 brick and concrete walls, 30 double-leaved wooden walls and 17 glass partitions, all measured in the laboratory. Some systematic experiments in the influence of the distance between the two leaves in a double wall show results which do not agree with those of Cammerer and Dürhammer.

Eight different floor constructions insulated against air-borne and impact sound were measured in one-family wooden houses and the influence of ordinary pugging studied. Similar experiments were carried out in brick and concrete buildings.

Public opinion concerning sound insulation in different types of flats was registered and interesting conclusions have been found. There seem to be rather different conditions in Sweden compared with England.

In addition, there are some general remarks—illustrated with results of measurements—about resonance in single walls, noise reduction in factories, the influence of noise on the quality of office work, and sound transmission through doors and windows. Various theories on sound transmission are also discussed.

Fritz Ingerslev, Denmark

1168. Hans Braumann, Medium reaction and acoustic radiation damping for a circular membrane (in German), Z. Naturforschung 3a, no. 6, 340–350 (June 1948).

The author studies the acoustic radiation of the freely vibrating circular disk. The wave equation is separated in oblate spheroidal coordinates [the author terms them bipolar coordinates]. He derives approximate expressions for the inertia increase and the energy radiation of the disk. [Actually these are the first terms of the rigorous expressions developed in ascending powers of the wave number.] Both the author's method and his results are not new; more accurate results are available. The author is apparently unacquainted with the work of, e.g., Kotani [Proc. Phys.-Math. Soc. Japan 15, no. 3, 30–57 (1933)], Hanson [Philos. Trans. roy. Soc. London 232, Ser. A, 223–283 (1933)], King [Proc. roy. Soc. London 153, Ser. A, 1–16, 17–40 (1935)], Bouwkamp [Groningen, thesis, 1941], and Sommerfeld [Ann. Physik 42, no. 5, 389–420 (1943)].

Courtesy of Mathematical Reviews C. J. Bouwkamp, Germany

1169. A. van der Ziel, Method of measurement of noise ratios and noise factors, Philips Res. Rep. 2, no. 5, 321–330 (1947).

In this paper methods are described as applied in this laboratory for the measurement of noise ratios of impedances and of the noise factor of receivers. Instead of standard signal generators, saturated diodes are used as standard noise generators. The noise voltages are amplified in a linear amplifier having a relatively small bandwidth (50–100 kc/sec) and detected by a thermocouple. The results obtained by these methods will be dealt with in subsequent papers. Author's summary

Ballistics, Detonics (Explosions)

1170. M. Garnier, Nadir trajectories. Pseudo-uniform paths (in French), Mémor. Artill. fr. 23, no. 2, 387–417 (1949).

The author devises a special procedure for calculating that portion of a vertical trajectory of a missile in which the acceleration is close to zero. This condition occurs very often for a projectile fired downward, during the late part of its flight. He justifies the effort involved in deriving and using this procedure on the ground that other methods may not give too accurate results for this portion of the trajectory. H. Polachek, USA

1171. M. A. Billard, Note on a graphical method in interior ballistics based on the use of the form function (in French), Mémor. Artill. fr. 22, no. 85, 619–648 (1948).

1172. John P. Vinti and Sidney Kravitz, Tables for the Pidduck-Kent special solutions for the motion of the powder gas in a gun, Ball. res. Lab. Rep. no. 693, 46 pp. (Jan. 1949).

This paper deals with an improvement of the Pidduck-Kent solution of the so-called Lagrange problem. The latter concerns the motion of the gases in the barrel behind the projectile. In this improved derivation, Abel's form of equation of state is used for the powder gases. The deviation is based on several simplifying assumptions (such as that the process is adiabatic and gas friction is negligible). Expressions are given for the ratios of pressure at the breech to pressure at the base of the projectile, and space mean pressure to pressure at base of projectile. Methods of calculation and tabulation are described.

A. D. Kafadar, USA

1173. Charles T. Johnson, Damage-distance relations for thin steel diaphragms 20 inches in diameter subjected to noncontact underwater explosions, David Taylor Model Basin Rep. no. 611, 15 pp. (Apr. 1949).

Underwater-explosion experiments have been employed to test the damage-distance relations for furniture-steel diaphragms 20 in. in diam, mounted in a caisson forming part of a smooth ver-

tical wall. The critical distances for rupture of diaphragms of about 0.105 in. thick have been determined, the logarithm of the critical distance varying linearly with the logarithm of the charge weight. The streak pictures of the early stages of the motion indicate that the initial velocity is in agreement with that predicted by free-plate theory, and that at the critical distance for rupture substantially all of the damage is caused by the shock wave, the gas bubble making only a minor contribution to the damage. A series of tests of bubble diaphragm assistance was made, yielding information on the mechanism of failure in underwater-explosions attacks against simple structures.

Stuart R. Brinkley, Jr., USA

Soil Mechanics, Seepage

(See also Revs. 1052, 1124)

1174. A. N. Puri, Soils: their physics and chemistry, New York, Reinhold Publishing Corp., 1949, xv + 550 pp. Cloth, 9.5 × 6.5 in., $7.

The book is written for students of agriculture, primarily beginners. Facts are stated and theories are left in the background in order not to confuse the novice. Controversial topics are avoided as far as possible, but no vacuum is left since the author makes his own stand known quite definitely.

Many modern texts represent adroit compilations, often undigested and without character, of data, theories and hypotheses selected in a way to assure largest possible use as a text by including the work, whether important or not, of most of the teachers in the field. This text is refreshingly different. The author has an excellent background in classical chemistry and physics, and has applied this background with tireless energy to the study of soils with respect to agricultural problems, especially of India. Practically all the data employed represent the fruits of the author's own experiments. The opinions stated are backed and rounded out by the author's own scientific work and practical experience. This is a strongly personal book, and the beginners in agricultural soil science can hardly find a better guide.

The book is composed of three parts: I. Chemistry of the soil; II. Mechanical analysis of soils; III. Soil moisture. The first part is excellent; the second part goes perhaps into greater detail than warranted by the limited practical importance of mechanical analysis and by the still very controversial state of the basic assumptions underlying all methods in this field. The soil-moisture part shows to what extent classical theory can be utilized in explaining and practically overcoming soil-moisture problems.

The reviewer, though himself an exponent of the "film-water" in contrast to the classical "capillary-water" concept, is reasonably certain that the agricultural student, considering his background and problems, will do better in following the example of the author than to risk the danger of getting lost in the greater complexity of the modern physicochemical concepts of soil water which had to be developed for the solution of soil-moisture problems in highway engineering.

Hans F. Winterkorn, USA

1175. Ralph E. Fadum, Concerning the physical properties of clays (in English), Proc. Sec. int. Conf. Soil Mech. Found. Engng. 1, 250–254 (1948).

The data reported in this paper were obtained during the construction of several large buildings in Boston. Field loading tests are compared with unconfined compressive-strength tests. Data are given to show how the natural water content of a homogeneous soil may be expected to vary with depth. Consolidation data are given for three different soil-sampling techniques.

Rollie G. Fehrmann, USA

1176. Wolmar Fellenius, Earth-statical computations with friction and cohesion (adhesion) under the assumption of circular-cylindrical slide surfaces (Erdstatische Berechnungen mit Reibung und Kohäsion (Adhäsion) und unter Annahme Kreiszylindrischer Gleitflächen), 4th ed., Berlin, Wilhelm Ernst & Sohn, 1948, 48 pp., 38 figs. Paper, 6 × 9$^1/_2$ in., DM 4.40 ($1.06).

The first edition of this classic book appeared in 1926. It gave the first comprehensive treatment of what is now generally known as the widely used Swedish method of investigating the stability of slopes and of vertical unsupported cuts in cohesive soils. Both cylindrical and plane surfaces of failure were investigated theoretically, and the results were summarized in graphical and in tabular form.

The present edition reproduces the original material with some additions. In this reviewer's opinion, the book, written in German by its Swedish author, still is a valuable reference for all engineers dealing with this type of problem. It is, however, to be regretted that more recent findings concerning the weakening effects of tensile stresses and the resulting cracks in the upper soil zone behind the top of a slope or of a cut, have not been considered in the present edition. Some of the numerical values given are therefore somewhat on the unsafe side, but can be corrected by users of the book for the above effects, if this point is kept in mind.

Gregory P. Tschebotarioff, USA

1177. József Jáky, Novel theory of earth pressure, Proc. Sec. int. Conf. Soil Mech. Found. Engng. 3, 65–69 (June 1948).

In Coulomb's theory the point of application of the earth pressure is arbitrarily assumed at one-third of the height, in accordance with hydrostatic distribution, and many attempts have been made to prove that the effective position of this point is higher. In this article the author gives a new theory which introduces the assumption that on vertical sections the earth-pressure line passes through the intersection of the sliding plane with the earth level, besides Coulomb's fundamental assumption of the plane sliding surface. The consideration of the equilibrium of a vertical slice leads to a differential equation for the stress distribution along the sliding plane. The integration in the case of a free foot of the sliding surface—as it occurs in timbered cuts—is given, and the conclusion is that the point of attack of the earth pressure in this case is effectively higher than the height given by the hydrostatic distribution.

Aurel A. Beleş, Rumania

1178. C. Torre, State of stresses in a heavy soil mass, Proc. Sec. int. Conf. Soil Mech. Found. Engng. 3, 57–61 (June 1948).

The author discusses the state of stresses in a semi-infinite mass consisting of heavy cohesionless grains of approximately equal size. As boundary condition he uses the involute parabola set up around the Mohr circles.

For an unloaded surface the stresses depend on the depth y and the grain size $h \cdot y = (z^2 - 1)h$ where $z = \sqrt{(1 + \gamma \cdot p)}$ and $2p = \sigma_n$, the equation of the involute parabola being $\tau^2 = 2p$.

A defect of this calculation is that the state of stresses is almost like that of the hydrostatical pressure. The shearing lines have the advantage of intersection at an angle which at increasing depth the shearing lines approach the straight lines, which form an angle of 45 deg with the horizontal ground surface.

Tabulated values for z and for y are also given in the article.

Ch. Széchy, Hungary

1179. F. L. Cassel, Slips in fissured clay, Proc. Sec. int. Conf. Soil Mech. Found. Engng. 2, 46–50 (1948).

The author shows that computation based on circular-slip theory, and the evaluation of its results on the basis of tests on undisturbed samples, does not provide sufficient information for

the determination of factors of safety of a cutting designed in a fissured clay. The deterioration of such cuttings originates according to numerous observations reported in the article, mostly in the zone of fluctuating ground-water levels. Two types of failure in two different groups of clays are distinguished. In both groups of clays the most effective and economically feasible safety measure against deterioration consists, in the author's view, in careful arrangement of drainage, for which specific suggestions are given.

P. Neményi, USA

1180. **L. J. Murdock, Consolidation tests on soils containing stones,** Proc. Sec. int. Conf. Soil Mech. Found. Engng. 1, 169–173 (1948).

Exploratory consolidation tests on remolded clayey soil containing stones are reported in this paper. The results seem to indicate that the estimates of the amount of consolidation and its time rate based on the fraction of fines alone are likely to be in error.

Consolidation tests of usual type on thick samples involve large side friction. This may be avoided by a special arrangement using triaxial compression.

Alexander Hrennikoff, Canada

1181. **Étienne Robert, Generalization of the Caquot formula for the calculation of foundations** (in French), Ann. Ponts Chauss. 118, 481–524 (July–Aug. 1948).

The writer gives a general formula for the maximum load of a foundation before rupture, based on the shearing resistance of the soil, expressed in terms of the angle of friction φ.

Prandtl gave the solution of the problem of the bearing capacity of plastic media, loaded at their surface and perpendicular to it. Caquot and Keverling Buisman applied the Prandtl solution to soils, and extended it to the case of the soil surface loaded next to the foundation, the soil being either cohesive or noncohesive (for cohesive soils they increased the normal stress by a term cohesion/$tg\ \varphi$).

The writer has introduced the following variables: (a) the angle between the loaded area and the horizon; (b) the angle between the perpendicular on the loaded area and the stress on this area; and (c) the angle between the soil surface next to the foundation with the horizon.

Earlier investigators have kept these variables constant at 0 deg, 90 deg and 0 deg respectively. The writer shows the influence of each of these variables, tabulates them, for different angles of friction, and provides illustrative examples.

In conclusion, the author points out that values for the bearing capacity of foundations obtained with his own and other formulas of this kind, strongly depend on the angle of friction which, therefore, should be determined with great care. He hopes that the results of calculations based on the proposed formula will be compared with experience.

F. C. de Nie, Holland

1182. **G. G. Meyerhof, An investigation of the bearing capacity of shallow footings on dry sand** (in English), Proc. Sec. int. Conf. Soil Mech. Found. Engng. 1, 237–243 (1948).

The bearing capacity of footings located on dry sand at depths reaching to six times the footing width is investigated by means of small-scale loading tests and compared with the theoretical estimations.

For footings resting on the surface, the tests show that the bearing capacity agrees fairly well with the theoretical value, while for footings situated at a shallow depth the bearing capacity is three times greater than the values estimated by theoretical considerations neglecting the shear strength of the overburden. However, a good correction can be obtained by including the friction of the footing on the sand, and the shear strength of the failure surface developed in the sand mass.

Aurel A. Beleş, Rumania

1183. **H. W. W. Pollitt, W. A. Lewis and P. H. T. Lewis, A laboratory study of the settlement of loaded rectangular plates into soft soil,** Proc. Sec. int. Conf. Soil Mech. Found. Engng. 3, 172–183 (June 1948).

The authors have attempted to study the behavior of track-laying vehicles, such as tanks and crawler tractors, by measuring the settlement of variously designed plates under load. The inertia of the loading device, the remolding of the two types of soils studied, the fact that no horizontal shear nor vibration were introduced in the test to reproduce actual conditions seem, to this reviewer, to lower very much the value of the results.

Robert Quintal, Canada

1184. **George W. Glick, Influence of soft ground on the design of long piles,** Proc. Sec. int. Conf. Soil Mech. Found. Engng. 4, 84–88 (June 1948).

The author discusses briefly old theories of the buckling strength of point-bearing long piles. Following Hjalmar Granholm [*The stability of piles surrounded by a supporting medium*, Stockholm 1929], he introduces the "coefficient of lateral displacement." He derives this value from the pressure vs. void-ratio curve, and develops methods for the estimation of buckling strength of point-bearing long piles. The steps of this method are: (1) laboratory determination of soil characteristics; (2) determination of coefficient of lateral displacement; (3) determination of critical load corresponding to pile length; (4) determination of ultimate strength based on initial distortion.

He also compares his computations with some loading tests which demonstrated that even soft muds provide a lateral support against buckling.

He finally arrives at the following conclusions: (1) lateral stability provided by the soil has a marked influence on the strength of long piles; (2) initial distortion of the piles contributes to loss of pile strength; (3) ordinary column formulas do not provide a measure of strength of long piles; (4) in making load tests of piles, consideration should be given to time element of consolidation; (5) where soil characteristics are known, a fair estimate of the ultimate strength of the pile may be determined by the methods outlined.

Ch. Széchy, Hungary

1185. **F. C. de Nie, Undulation of railway embankments on soft sub-soil during passing of trains,** Proc. Sec. int. Conf. Soil Mech. Found. Engng. 2, 8–12 (1948).

The author divides the causes of track settlement into four main groups: consolidation, plastic flow, penetration of the ballast into the roadbed, and undulation. The last is an instantaneous deformation with complete rebound so that no displacement of the solid, liquid or gaseous elements of the soil must take place. Measurements of this phenomenon were made by anchoring a pile at a depth where no movement can take place, and inscribing mechanically the track settlement as a train passes by. The results of a number of tests thus performed at a few points between Gouda and Oudewater have led the author to some suggestions regarding train velocity and axle distribution under the rolling stock, as well as roadbed design on soils which will give rise to this problem. A suggestion is also made regarding the construction of a mobile rig which would create a pulsating load at any point along the track.

Robert Quintal, Canada

1186. C. van der Poel, Dynamic testing of pavements and base courses, Proc. Sec. int. Conf. Soil Mech. Found. Engng. 4, 157–163 (June 1948).

Three types of airfield runways were subjected to mechanical vibration at frequencies from 10 to 40 cycles per sec. The rate of wave propagation and vibration amplitude was measured at various distances from the disturbance. The former was found to have different values over certain frequency ranges; rates of 114 and 140 meters per sec being related respectively to transverse waves in subsoil and base course, and 200 meters per sec to bending vibration of the concrete slab in the sandwich construction. Approximate pavement-stress calculations are deduced from the observed amplitudes, and comparison made between two types of construction. R. N. Arnold, Scotland

1187. A. Casagrande, Soil mechanics in the design and construction of the Logan airport, Harvard Univ. Publ. grad. School reprint no. 467, 16 pp. (1948-9) = J. Boston Soc. civ. Engrs. 36, 192–221 (1949).

According to this paper, the main problem at the Boston airport, as far as soil mechanics was concerned, was the evaluation of the borrow material. Prior to construction, the soil profile was roughly composed of five to ten feet of organic silt underlain by a layer of sand, then up to one 100 feet of blue clay underlain by sand and gravel. It was proposed to build up this area along the runways with 20 feet of hydraulic clay dredged from adjacent harbor areas and pumped to the site. The mean low-water level was approximately at the top of the natural soil profile.

The lack of knowledge of the properties of hydraulic clay fills would have warranted extensive tests, but economic considerations forced the designers to take calculated risks. In view of this, a number of tests were carried out, and the curve extrapolated with the intention of checking it at various intervals.

The surface strength of the fill depending on its consolidation, a measure of this property was required, but the usual method through surface settlements was ruled out because of the impossibility of finding which proportion of the total settlement came from each material in the profile. Pore-pressure measurements were thus proposed and carried out after a sensitive and fairly foolproof piezometer was developed, detailed specifications being given in an appendix.

Pore-pressure vs. depth curves were traced for various time increments and correlated with the observed combined settlement-vs-time curve. It thus became possible to estimate the total settlement, the time required for ultimate consolidation and also the increase in strength of the upper crust. Information was also collected which may permit the design of subsequent runways such that differential settlements will not occur at crossings between old and new structures. Robert Quintal, Canada

Geophysics, Meteorology, Oceanography

(**See also Rev. 1046**)

1188. Peter Sieber, Vertical motions and atmospheric pressure variations (in German), Met. Rdsch. 1, 322–329 (May/June 1948).

The author derives an approximate formula relating the vertical component of the wind with the horizontal divergence of the deviation of the actual from the geostrophic wind, and applies it to qualitative explanation of observed vertical motions. On the basis of several hypotheses regarding advection and an empirical formula, he then constructs an expression giving the time-rate of change of the surface pressure in terms of meteorological variables. He gives a graphical example to show how this result can be applied to the prediction of future pressure distributions from weather-map data. C. A. Truesdell, USA

1189. Heinz Lettau, On weather-producing vertical motions and their determination from the pressure field (in German), Met. Rdsch. 1, 257–260 (Mar./Apr. 1948).

By weather-producing vertical motions the author means those gradual motions which prevail over areas several hundred kilometers on a side. By use of the continuity equation and the equations of motion expressing a quasi equilibrium between friction, pressure force, and deflective force, he derives the well-known result that, under these conditions, vertical motions depend upon the departure of the wind from geostrophic.

The author extends previous work, however, by including internal friction in his equation for the vertical velocity (in the form of an "exchange" coefficient which is independent of the height) and by expressing his relation between the vertical velocity and the pressure field in a form which can be evaluated from the weather chart.

A sample evaluation by his method shows good qualitative agreement with another more direct method of calculating vertical motions. It must be pointed out, however, that a perhaps severe limitation to the prediction of vertical motions by the author's method lies in his assumption of a quasi-stationary situation, i.e., that the acceleration term only balances the frictional term. With a strongly accelerated field of motion, his results can hardly be expected to apply.

Joanne Starr Malkus, USA

1190. Heinz Lettau, Maxima of wind velocity as function of the earth's rotation (in German), Met. Rdsch. 1, 451–453 (Sept./Oct. 1948).

Continuing his previous paper [Met. Rdsch. 1, 99–100 (1947); see Rev. 2, 1212], the author shows that a limiting value exists for geostrophic stationary flow in the atmosphere, which is found to be about $0.104\ A \sin \varphi$ km/h, where A is the distance between a coherent high and low in km. The results of this paper should also be applicable in the case of other celestial bodies.

Horst Merbt, Germany

1191. J. M. Austin, Temperature advection and pressure changes, J. Met. 6, no. 5, 358–360 (1949).

The origin of pressure changes at sea level is discussed in the light of temperature-advection theories. From empirical data certain relationships between these pressure changes and the advection at low levels are found by correlating the pressure changes with temperature advection near isallobaric centers calculated by means of geostrophic flow. No conclusions can be drawn with regard to the mechanism and the causes of these changes.

Horst Merbt, Germany

1192. Merrill Bernard, Precipitation, in ***Hydrology,*** ed. by O. E. Meinzer, New York, Dover Publications, 1949, pp. 32–55.

An expository popular article. Ed.

1193. J. G. Scholte, On the large displacements commonly regarded as caused by Love-waves and similar dispersive surface-waves: I, II, III, IV, Proc. Nederl. Akad. Wetensch. 51: 533–543, 642–649, 828–835, 969–976 (1948).

In this paper an attempt has been made to calculate the motion caused by a disturbance taking place in a stratified medium. The method used in a previous paper on this subject started with a Fourier analysis of the time factor of the initial disturbance; the motion connected with each component of the Fourier series was calculated and the final result was reached by adding up these partial movements. This method, however, yields results which are not entirely interpretable. The first difficulty one meets when trying to correlate these results to the observed

properties of seismograms lies in the fact that the time of arrival of the surface waves cannot be calculated; as these waves form the major part of the movement at large distances this impossibility is very disconcerting. A second difficulty is scarcely less important: as these surface waves satisfy the frequency equation of the system (for instance the Love equation) their phase-velocity is a function of the frequency; it follows that the surface waves have to show the usual dispersion properties. But the observed relation between the velocity and the period of the large displacements is in direct contradiction to the dispersion theory.

These considerations have led to an investigation of the surface waves, which is described in chapter I; it appears that it is by no means a priori certain that the surface waves which satisfy the frequency equation occur in seismic disturbances. Consequently it is possible that a method which obtains these waves as the predominant terms of a series of movements is not adequate to the problem. It is therefore necessary to use another method in order to ascertain whether these surface waves are relevant or only the results of an inadequate method. By means of the method of Carson it is possible to avoid the Fourier analysis of the initial disturbance; this method, applied in chapters II and V, leads to results which are easily interpreted as they contain no dispersive surface waves.

It is difficult to follow step by step the physical implications of the author's treatment. There are a number of typographical errors, but they are not troublesome.

In part III conditions for a purely longitudinal movement are assumed. The author refers to the displacements taking place under the postulated conditions as motion in an ocean caused by a subaqueous explosion, although he does not include the Ewing bubble effect, density-temperature channeling, and other phenomena characteristic of a physical ocean. The structure postulated by the author is a homogeneous liquid layer of finite thickness resting on a solid body of semi-infinite extent. The primary disturbance is first assumed to occur in a plane within the liquid layer parallel to the interface. The results are then transformed to correspond to a point disturbance within the liquid layer by a Laplace transformation applied to the displacement potential. A Stoneley function is found which determines a major part of the disturbance.

In the first paragraph of part IV the author supposes a disturbance in the liquid layer propagated downward into the second medium, and considers that cone of ray paths which will impinge on the intersurface at the critical angle or greater, so as to set up boundary and reflected waves. The corresponding Laplace functions are analyzed for a series of conditions and time intervals. The second paragraph begins with a statement concerning a supposed seismological definition of a surface which would be conceded by few, if any, seismologists. The paragraph is largely devoted to a discussion of Cagniard's theoretical work and of supposed results from observation.

Courtesy of Mathematical Reviews J. B. Macelwane, USA

1194. **J. G. Scholte, On true and pseudo Rayleigh waves,** Proc. Nederl. Akad. Wetensch. 52, 652–653 (1949).

The author discusses solutions by himself, in previously published papers, and by Cagniard [*Réflexion et refraction des ondes seismiques progressives*, Gauthier-Villars, Paris, 1939] for the formation and propagation of boundary waves of the Rayleigh and Stoneley type. He states that his previous criticism of the root of the equation of condition $D = 0$ obtained by Cagniard was not justified, and that there are really two roots, one of which is very important when one of the two semi-infinite media is a fluid of low density like the atmosphere, and the other is solid, while the second root is the important one when both are solid or one is solid and the other is liquid like the ocean. The first was obtained by Cagniard and the second by the author.

Courtesy of Mathematical Reviews J. B. Macelwane, USA

Lubrication; Bearings; Wear

(See also Rev. 1111)

1195. **Milton C. Shaw and E. Fred Macks, Analysis and lubrication of bearings,** New York, Toronto, and London, McGraw-Hill Book Co., Inc., 1949, xv + 618 pp., 375 figs. Cloth, 9.2 × 6.3 in., \$10.

The book is a very valuable compendium of information covering the whole field of the subject. The first four chapters may be regarded as being somewhat distinct from the remainder of the book because they deal with more specialized applications of bearings to internal-combustion engines, particularly aircraft engines, and are mainly concerned with methods of estimating the loading applied. The subject of lubrication proper is dealt with in the remaining chapters in a very comprehensive manner, the chapters on dynamically loaded gearings and high-speed considerations being noteworthy. The authors have summarized most important contributions to the subject selected from a wide international field and present the essentials in a coincise yet readable manner. In addition to reviewing work of others, they include some of their own work, for example, the discussion of the hydrosphere and the full-floating bearing. The emphasis given to output of different workers must always be a matter of opinion, but the authors display fairness and judgment in their selection. Not only are theoretical aspects of the subject discussed but the authors pay considerable attention to the more modern experimental methods such as electron microscopy. Whereas the first four chapters are likely to be of interest in a restricted field, the book as a whole will certainly be recognized as a reference work invaluable to all those who wish to be up to date on the subject of lubrication.

F. T. Barwell, Great Britain

1196. **A. Buske and W. Rolli, Measurement of oil-film pressures in journal bearings under constant and variable loads,** Nat. adv. Comm. Aero. tech. Memo no. 1200, 43 pp. (1949).

This is a translation of a 1937 German publication describing an oil-film pressure recorder and giving results of a number of measurements made with it.

The first part of the report describes the instrument which is noted for its small size and rapid rate of response which make possible the measurement of average pressures over extremely small areas, and the accurate measurement of rapidly varying loads. The second part of the report deals with the measurements themselves which range up to peaks 200 or 300 atm or more, and with periodic loads of frequency between 750 and 2200 per min.

F. J. Maginniss, USA

Marine Engineering Problems

(See also Revs. 1086, 1125, 1126, 1127, 1139, 1151)

1197. **John McCallum, An examination of the pressure distribution over a model hull,** Trans. Instn. nav. Archit. Lond. 90, 316–333 (1948).

This paper describes tests carried out on a ship model in a wind tunnel with the primary object of investigating the pressures acting on the model hull of a given wind speed. On the port side of the model $^{3}/_{4}$-in-diam holes, numbering about 200, were drilled over the bottom and side shell. The shell lines were carried well above the experimental-load water line in the hope that boundary

effects would be eliminated as far as possible. (A double inverted model was not used in view of the desirability of having as large a Reynolds number as possible.) The apparatus employed for measuring pressures was the common micromanometer. A diagram shows the variation of the normal pressure on the model. A surprising result is the absence of any positive pressure change around the stern. An attempt has also been made to estimate the form resistance and frictional resistance of a similar full-scale vessel in water, but this attempt was a secondary consideration.

The pressure resistance for the model may be represented by $\int p_l ds$, where p_l is the longitudinal component of the pressure. The double integration gives a figure for the net longitudinal thrust exerted on the model. The pressure resistance of the full-scale similar ship is then found with the condition that each model must be run at the same Reynolds number. The values of the effective horsepower (*EHP*) to overcome pressure resistance are tabulated for various speeds together with the corresponding figures for the residuary *EHP* found by deducting the frictional *EHP* from the total *EHP* derived from the tank results. The most striking discrepancy of this comparison is that until wave making becomes pronounced, the pressure curve runs well above the residuary. In order to take into account the variation of the roughness, the author has to turn to aerodynamical methods involving the use of the frictional coefficient C_f.

The paper is followed by very interesting remarks of W. C. S. Wigley, G. S. Baker and others. Wigley calculated the wave resistance of the model and has determined the amount of *EHP* which is not accounted for either by the frictional resistance or by the calculated wave resistance. He found that the pressure resistance as estimated by the author is far too high, and he attributes this chiefly to the neglect of the boundary condition at the free surface. Baker remarks that the second part of the paper (comparison with the resistance of a full-scale vessel) is dubious, but he commends the first part. L. J. Tison, Belgium

1198. **F. H. Todd, The determination of frictional resistance,** David Taylor Model Basin Rep. no. 663, 7 pp., 2 figs. (March 1949).

The article represents the comments by the author completing his introductory remarks on the subject of skin friction read before the Fifth Int. Conf. of Ship Tank Superintendents in London, Sept. 14–17, 1948, as introduction to and part of the discussion.

The author gives here a very able review of and valuable comments on the different formulas proposed for the determination of the frictional resistance of ships. He considers especially the merits of the Schoenherr formula and the allowance hereto proposed by the American Towing Tank Conference (1947), versus that of Froude as modified by the Paris conference of 1935. The author, in the name of the David Taylor Model Basin delegates, submits the proposal to adopt the Schoenherr mean line as basis for the analyses of ship-model tests. Einar Hogner, Sweden

1199. **L. Landweber, Effect of transverse curvature on frictional resistance,** David Taylor Model Basin Rep. no. 689, 8 pp. (March 1949).

Prediction of the total drag of prototype ships from model tests depends upon the accuracy with which the measured model drag can be resolved into frictional resistance and so-called residual (i.e., wave and form) resistance. Since the frictional resistance must be calculated from boundary-layer formulas based upon the drag of flat plates, errors may thus arise from the essential difference in boundary-layer development along flat and curved surfaces. This paper seeks to evaluate the magnitude of the error due to curvature in the transverse plane. The analysis is based upon the combination of four equations customarily used in boundary-layer computations: the empirical Blasius relationship for wall shear, the $^1/_7$-power law for the velocity distribution, the equilibrium equation between shear and drag for a circular cylinder, and the expression for drag in terms of the momentum integral. The resulting relationships permit calculation of the relative reduction in boundary-layer thickness and the relative increase of both local shear and total drag with increasing ratio of boundary-layer thickness to radius of transverse curvature. Tabular and plotted values of the functions and an illustrative example of their application are included.

Hunter Rouse, USA

1200. **Roger Brard and Jean Bleuzen, The present state of research on gyration at the Paris model basin** (in French), Bull. Ass. tech. Marit. Aéro. 46, 271–340 (1947).

The purpose of the investigation is (1) to explain why sometimes gyration experiments on ship models may reveal different regimes of flow at the rudder for otherwise identical conditions; (2) to give a general review on the gyration research at the Paris Tank.

Rudder forces and conditions of flow have been investigated experimentally for:

(1) Isolated rudders; (2) rudders behind a plot plate picturing the longitudinal center plane of models; (3) rudders behind models, on a straight course and on a circular path.

The following results are stated: (1) The ratio of diameter-of-gyration to length-of-vessel for the model can exceed the corresponding one for the ship by 10–15% (cavitation excluded). (2) The difference in the regime of flow mentioned is due to separation. Special studies are made as to the influence of the rudder type on separation. (3) The influence of the hull on the rudder is appreciable. (4) The influence of the propellers for the special arrangements tested was only secondary. Numerous graphs illustrate the results. Georg P. Weinblum, USA

1201. **T. Thorpe and K. P. Farrell, Permanent moorings,** Trans. Instn. nav. Archit. Lond. 90, 111–153 (1948).

This is a report on investigations undertaken to develop mooring designs having greater holding power and more economical construction than the present well-established types. An analytical and experimental approach to the problem is undertaken and a vastly improved design is indicated. This involves a utilization of the energy available in the sag of the mooring cable, to allow a gradual snubbing of the ship. Experimental data useful in the design of moorings are given. Excellent information is presented on a subject concerning which little data are available.

F. E. Reed, USA